S100 1 and 2

THE OPEN UNIVERSITY

Science Foundation Course Units 1 and 2

Science: Its origins, scales and limitations

Observation and measurement

The Open University

Science Foundation Course Unit 2

OBSERVATION AND MEASUREMENT

Prepared by the Science Foundation Course Team

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Contents

Table A

A List of Scientific Terms, Concepts and Principles used in Unit 2*

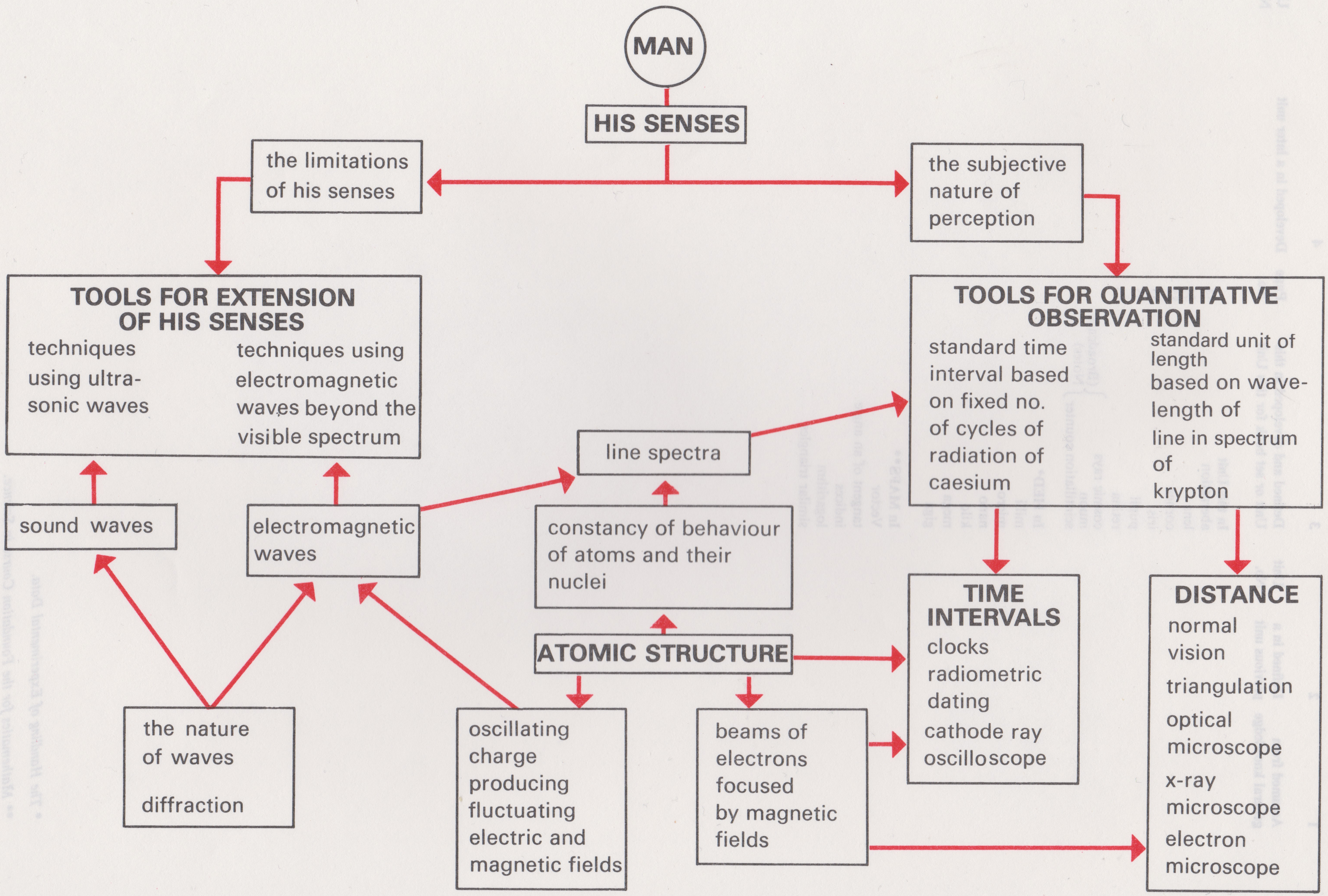
Taken as pre-requisites			Introduced in this Unit			
1	2		3		4	
Assumed from general knowledge	**Defined in a previous unit**	**Unit No.**	**Defined and developed in this Unit or set book for this Unit**	**Page No.**	**Developed in a later unit**	**Unit No.**
			In this Unit			
principle, rule	law	1	frequency	11		
			source of waves	11		
			amplitude of waves	12		
			velocity	12	velocity	3
			wavelength	12		
			ultrasonic wave	13	wave nature of electromagnetic radiation	28
			atom	15		
			electric charge	15		
			electromagnetic radiation arising from movement of electrical charge	15	dependence of resolving power on wavelength, diffraction	28
			electron	15	constancy of atomic structure	30
			field	15		
			nucleus	15	atomic nature of all matter	5, 6
			electromagnetic waves	19		
			spectrum	19	radiometric dating	23
			infra-red, ultra-violet, and X-ray radiation	20		
			high-speed cinematography	22	line spectrum	6
			time-lapse photography	22		
			subjective nature of perception	23	nucleus	31
			units	25		
			units are pre-requisite of measurement	25	field	4
			metre	26		
			element	27	element	6
			line spectrum	27		
			mean solar day	28	compound	8
			second	28	refraction	22
			base-line	31		
			triangulation	31	electric charge	4
			diffraction	33		
			resolving power	34		
			battery	36		
			conduction electrons	36		
			electric circuit	36		
			electron microscope	37		
			conservation of electric charge	40		
			half-life	40		
			radioactivity	40		
			exponential function	42		
			cathode-ray tube	45		
			time-base	46		
			compound	47		
			developer	47		
			latent image	47		
			photographic emulsion	47		
			fixer	48		
			compound optical microscope	49		
			objective lens	49		
			magnifying glass	49		
			focal length and focal point of a lens	50		
			eyepiece lens	51		
			image	53		
			refraction	53		

* *Any scientific terms used in this Unit but not listed are marked thus † and defined in the glossary (Appendix 7, p. 59).*

1 Assumed from general knowledge	2 Defined in a previous unit	Unit No.	3 Defined and developed in this Unit or set book for this Unit	Page No.	4 Developed in a later unit	Unit No.
			In this Unit			
			aberration	54		
			lens	54		
			cornea	55		
			iris	55		
			pupil	55		
			retina	55		
			cosmic rays muon scintillation counter } (Broadcast Notes)			
			In HED*			
			milli			
			micro			
			nano			
			kilo			
			mega			
			giga			
			In MAFS**			
			Vector			
			tangent of an angle			
			indices			
			logarithm			
			similar triangles			

* *The Handling of Experimental Data.*

** *Mathematics for the Foundation Course in Science.*


MAN
HIS SENSES
the limitations of his senses
the subjective nature of perception
TOOLS FOR EXTENSION OF HIS SENSES
techniques using ultra-sonic waves
techniques using electromagnetic waves beyond the visible spectrum
TOOLS FOR QUANTITATIVE OBSERVATION
standard time interval based on fixed no. of cycles of radiation of caesium
standard unit of length based on wave-length of line in spectrum of krypton
sound waves
electromagnetic waves
line spectra
constancy of behaviour of atoms and their nuclei
ATOMIC STRUCTURE
TIME INTERVALS
clocks
radiometric dating
cathode ray oscilloscope
DISTANCE
normal vision
triangulation
optical microscope
x-ray microscope
electron microscope
the nature of waves
diffraction
oscillating charge producing fluctuating electric and magnetic fields
beams of electrons focused by magnetic fields

Objectives

When you have completed the work for this Unit, you should be able to:

1. Define, or recognize adequate definitions of, or distinguish between true and false statements concerning each of the terms, concepts and principles in column 3 of Table A.

2. Given simple unlabelled diagrams, assign names and functional descriptions to the main parts of the following:

—the human eye
—a compound optical microscope
—an electron microscope
—a cathode-ray oscilloscope.

3. Draw and explain simple ray diagrams to show how the image of an object is focused by:

—the human eye
—a compound optical microscope
—an electron microscope.

4. Give or recognize examples of how man has extended his senses by using:

—ultrasonic sound waves
—infra-red rays
—ultra-violet rays
—X-rays.

5. State or recognize the approximate ranges of frequency and wavelength over which the following electromagnetic rays extend:

—radio
—TV
—infra-red
—visible
—ultra-violet
—X-ray.

6. Describe how the electric charge within an atom is distributed between its electrons and nucleus. Explain briefly, or recognize, valid descriptions of why two pieces of material rubbed together may become capable of attracting one another.

7. Compare and contrast the mode of generation and propagation of sound waves and electromagnetic waves.

8. Describe the constitution of a photographic emulsion; explain briefly the formation of a latent image, the function of the developer and fixer, and how a final positive image is obtained from the negative image.

9. Name three examples of sensory receptors becoming fatigued by prolonged stimulation and describe the effects produced by such fatigue. Give one reason for believing that some visual illusions arise from the way the brain organizes the information sent to it by the senses.

10. Describe briefly how our system of units evolved and why earlier definitions of the units of length and time were considered unsatisfactory; give the present-day bases of the international standards of length and time.

11. Describe briefly under what conditions electromagnetic fields and waves are produced, and how their existence can be detected. Describe the directional orientation of the field at a point relative to the position and motion of the charge producing it.

12. Describe briefly the behaviour of conduction electrons in a metal when the metal is placed in an electric field, and why a battery is necessary for maintaining an electric field round a circuit. Describe briefly how one can produce a beam of free electrons.

13. State the law of conservation of electric charge.

14. Either by recalling the appropriate formula, or selecting from given formulae, and given relevant data, solve simple problems concerning:

—measurement of distances by triangulation
—the size of an image formed by a lens
—exponential variation
—the relationship between velocity, wavelength and frequency of waves.

Introduction

All living organisms depend to some extent on their ability to observe their surroundings. Man is no exception. His observations must, of course, come to him through his senses, but he has come to realize that his senses are limited, both in their scope and reliability. By the ingenious invention of suitable tools and techniques, however, he is to a large extent overcoming these limitations, and in so doing has opened up a rich new experience of the world in which he lives.

With these tools he has been able:

(a) to extend the range of his senses so as to detect sounds and light that otherwise would not be accessible to him;

(b) to quantify his observations and make them into reproducible measurements;

(c) to investigate phenomena occurring on scales of length and time that are either much greater or much smaller than those that come within his natural domain.

This then briefly is the theme of this Unit. It is a very broad one as you can well imagine, so naturally we have had to be very selective in our material. At first we thought we would take you to the very frontiers of, for example, small scale measurements. This turned out to be none too easy at this early stage of the Course. In any case we thought it would be more appropriate to deal with these techniques later when we discuss elementary particle physics.

The illustrations we eventually decided upon were chosen for a variety of reasons. For example, in the measurement of short time intervals we describe a cathode-ray oscilloscope. This is an instrument very similar in principle to your television set. Relying as we do at the Open University on television for so much of our teaching we felt you ought to know something of what goes on behind that screen. We shall also be making extensive use of photography as a means of communicating with you—both in the form of photographs issued with the correspondence texts and as films shown on television. This was one of the reasons why we chose to describe the photographic technique. Our decision to concentrate on the microscope as a means of measuring small distances was influenced by the fact that you were to be lent a microscope for your own use and so should obviously know how it works.

A wide-ranging topic like measurement also allows us to fill in a great deal of background knowledge. For example, we can cover topics such as the nature of sound, the nature of light, elementary electricity, simple optics, etc.

If you already have a firm grounding in basic physics, then you will find much of this Unit easy going. Don't miss any of it out, though, as we want you to follow through the general theme—but do feel free to take the material at a brisk pace. Providing, of course, you can manage the tests at the end of the Unit, you may then make an early start on Unit 3; this introduces Einstein's Special Theory of Relativity and it almost certainly will give you scope to stretch yourself. If on the other hand your scientific background is not so good, then you have a considerable amount of ground to cover this week. Don't let this deter you though—if you go about it conscientiously, taking careful note of the meaning of each new term as it is introduced, you will get through it perfectly well!

Section 1

2.1.1 Sounds Man cannot Hear, Light he cannot See

You saw in Unit 1 how important is the ability to make observations. Even simple organisms need to have some means of observing the surroundings. If these means were not available it would be impossible to locate food and avoid predators.

In man, this process of observation goes much further. Each individual can draw not only from his own personal experiences but also from that of others. He has access to a large accumulated body of information which is continually growing. This ever deeper insight into the workings of nature in turn leads to an increasing ability on man's part to manipulate the environment.

But much of the information he wishes to acquire from his surroundings is not directly accessible to his senses. There are sounds his ears cannot hear; there are forms of light his eyes cannot see.

We have already mentioned how he has developed ways of overcoming some of the limitations of his body; he fashions tools that develop his manipulative skills and others that increase his physical strength. We now consider others that add to his powers of observation—they are extensions to his senses.

2.2.1 The nature of sound

We begin with the sense of hearing. Hearing probably comes second only to sight in its importance as a means of observing the environment. It centres upon the ear's ability to detect sound waves, so we first describe the nature of sound.

Sound waves are generated by a moving body called a *source*. As it moves, it pushes against the surrounding medium (usually the air) and in so doing causes a disturbance. We can take as an example of a source a vibrating metal strip clamped at one end, as in Figure 1. Its to-and-fro motion sets the nearby layer of air particles moving.

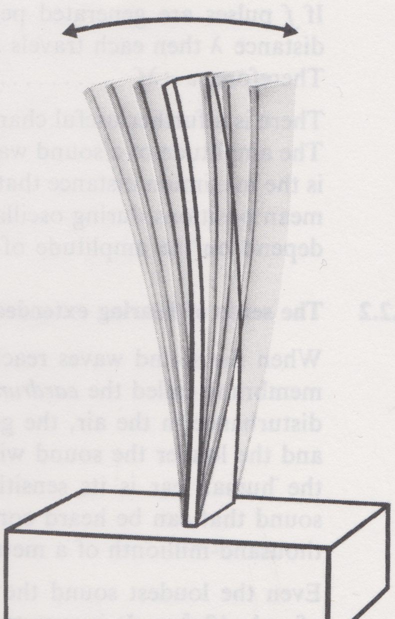


Figure 1

A source of sound—a vibrating metal strip.

What do you expect the particles in the nearby layer to do now:
(a) move right away from the source, passing through the rest of the medium;
or
(b) transfer the disturbance to the next layer?

They in their turn push against the next layer and rebound and so on. Like the metal strip itself, the individual air particles oscillate about their mean† position (the position they would occupy if the metal strip were not moving), while the disturbances initiated by the source progressively move outwards. The disturbances consist of regions where the *density* (the number of particles in a given volume) is successively greater and less than the mean density. A series of repeated disturbances is called a wave train. Figure 2 shows the positions of these disturbances at various times after the strip has been set vibrating.

The number of high density regions passing any given point in a given time is called the *frequency* of the wave train and we denote it by f. It has the same value as the number of complete to-and-fro motions of the source in the given time.

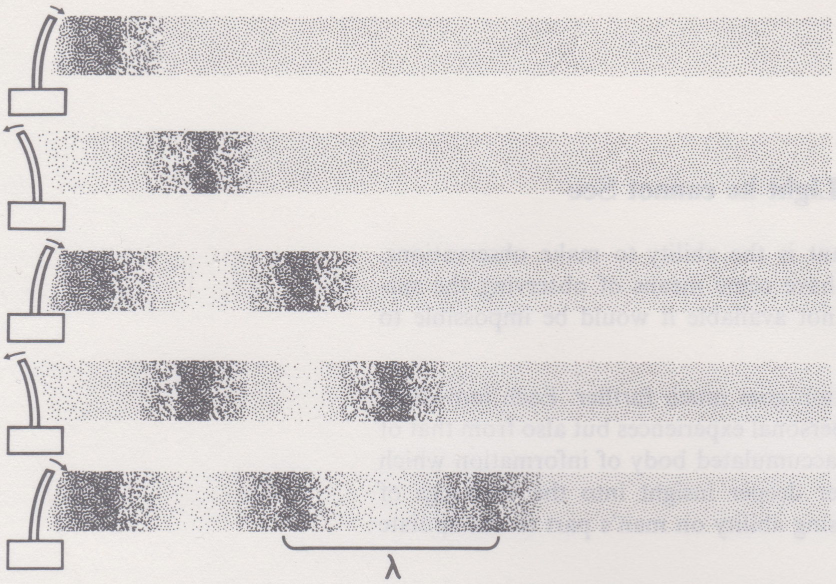


Figure 2

Diagram showing successive regions of high and low density being generated by a vibrating metal strip. The wavelength, λ, is illustrated on the final drawing.

If the vibration of the source is maintained at a steady rate, then it is clear that the distance between successive pulses of high density will always be the same. This constant distance is called the *wavelength* (Fig. 2) and is denoted by λ (a Greek letter pronounced 'lam-da'). It also represents the distance between successive regions of low density. The wavelength obviously depends upon the *velocity*, v, with which the pulses are moving away from the source (i.e. the distance they travel in a given time).

What relationship do you expect between f, v, and λ?

If f pulses are generated per second and they are each separated by a distance λ then each travels a distance λf per second.
Therefore, $v = \lambda f$ .... (1)

There is a further useful characteristic of a wave, and that is its *amplitude*. The amplitude of a sound wave is related to the loudness of the sound and is the maximum distance that the particles of the medium move from their mean positions during oscillation. In the case we are considering this will depend on the amplitude of the motion of the metal strip.

2.2.2 The sense of hearing extended

When the sound waves reach the ear, they set vibrating a thin stretched membrane called the *eardrum*. Naturally the greater the amplitude of the disturbance in the air, the greater will be the movement of the eardrum and the louder the sound will appear. One of the remarkable features of the human ear is its sensitivity to very small amplitudes. The faintest sound that can be heard corresponds to a displacement of one hundred-thousand-millionth of a metre* (i.e. 10^{-11} m)**.

Even the loudest sound the ear can tolerate has a maximum amplitude of only 10^{-5} m. It is easy to appreciate from these figures that the extraordinary sensitivity of the ear makes it exceedingly difficult for the designers of aircraft, for example, to keep the extraneous noise down to what might be considered a tolerable level.

The ear, of course, not only differentiates between faint and loud sounds, but also between sounds of different frequency. If the pulses arrive at the rate of about 20 cycles† per second or less, the ear is able to distinguish the separate pulses. Above this frequency the effect produced is that of a continuous note—the higher the frequency, the higher the note.

* *1 metre = 39.4 inches.*
** *The amplitude for the faintest audible sound depends upon the frequency involved. The figure quoted is for a frequency of 1 000 cycles per second.*

It is at this stage that one encounters a limitation of the ear. Above about 20 000 cycles per second, sounds can no longer be heard. The actual value of the frequency at which the cut-off occurs depends upon the individual and varies with age. Waves with frequencies greater than 20 000 cycles per second are classified as *ultrasonic waves*.

The ears of some animals respond to much higher frequencies than 20 000 cycles per second. This is why it is possible to have special high frequency whistles that produce waves audible to dogs, but not to humans. Thus man finds himself up against a barrier—a barrier imposed by a limitation of one of his senses. He has even to face the fact that some of the so-called lesser animals have a natural ability that excels his own in this respect! If he were to extend the range of his hearing to higher frequencies with an instrument of some sort, would it be to his advantage? One has only to consider the bat to find an answer (Fig. 3).

Figure 3
A bat in flight.

Bats have a remarkable ability to fly at speed in and out of trees in darkness. As they fly they generate ultrasonic waves. How the waves are generated is not fully understood, but it is believed that they radiate from the bat's nose during both inhalation and expiration. Thus the sound is emitted during the complete breathing cycle. The frequency generated is about 50 000 cycles per second and this corresponds to a wavelength of 0.7 cm. Sometimes the frequency may be as high as 150 000 cycles per second.

The bat issues short bursts of these ultrasonic waves in a well-defined forward direction. In normal cruising flight, the pulses are emitted at a rate of 5 to 20 bursts per second with silence in between. These pulses bounce back to the bat from any objects in its path (Fig. 4). Knowing the direction from which the echo comes, and deducing the distance from the time that has elapsed between the emission of the original burst and the arrival of the echo, the bat can locate the object.

Figure 4
Diagram illustrating the way bats emit short pulses of ultrasonic waves in a narrow forward cone. These are then reflected back to it from objects in its path.

But why use ultrasonics—why not ordinary sound? The reason is to be found in Figure 5. Here water waves are seen striking objects of various sizes. You can see that when the object has dimensions almost as small as the wavelength, the wave train is hardly disturbed at all. It is only when the size of the object is greater than a wavelength that it reflects an appreciable amount of the wave.

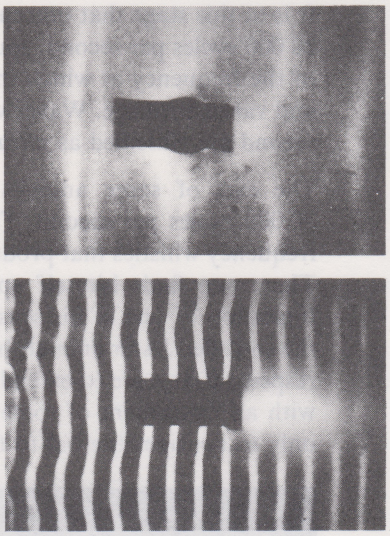


Figure 5

Water waves of different wavelengths advancing from the left and meeting an object. The smaller the wavelength the greater the disturbance caused to the wave train. Note there is an almost total absence of shadow in the upper photograph.

Waves of all kinds behave in similar ways. What is true of water waves is also true of sound. The size of the object to be detected with the sound waves sets the limit on the size of the wavelength. The bat uses a wavelength that is ideal for detecting branches, twigs, and leaves which can then be avoided, and insects in flight on which to prey. The wavelength is small enough to get good reflection but at the same time is not too small. Ultrasonics of shorter wavelength dissipate their energy and die out more rapidly, so there is an advantage in not having the wavelength any shorter than necessary. (You might like to know that some insects, mainly moths, have developed ears that are sensitive to the frequencies emitted by bats, and so they receive a warning of the bat's approach. Some moths are even capable of producing their own ultrasonic waves and these appear to deter the bat from coming too close.)

Man has now succeeded in copying very effectively the bats' technique. We can generate and detect ultrasonic waves, for example, by making crystals vibrate rapidly. Because of their dimensions and physical properties, these crystals have a natural tendency to vibrate much more quickly than any metal strip or eardrum. They provide the means of both generating and detecting ultrasonic waves, and can be combined into a system similar to that used by a bat. Such man-made systems are used at sea, for example, and help in the location of submarines and shoals of fish. They even distinguish between shoals of large and small fish. Systems are being developed for the guidance of the blind and these have now reached the stage where they can successfully be used to locate pencils at a distance of several feet.

Ultrasonic waves can also probe into objects that are opaque to light, such as metal castings. The waves can be passed through the object and detected on the far side. For normal castings there will be some characteristic amplitude for the transmitted wave. If however a specimen has any flaws or impurities in it, the waves will be partially reflected from them. This leads to a detectable reduction in the amplitude of the transmitted wave. An abnormally weak, transmitted wave is thus an indication that a specimen is faulty.

Section 3

2.3.1 The Nature of Electromagnetic Fields and Light

Impressive as these new skills may be, they compare hardly at all with the enormous developments that have stemmed from man's extension of that most important of all senses—sight.

The eye, like the ear, responds to the reception of a wave. But this wave is quite unlike that of sound. Sound needs a medium; without the air particles, or particles of some other gas, solid or liquid, there would be nothing to carry the vibrations from the source to the detector.

Light waves, however, need no medium. They travel across empty space—if they did not one would never see the Sun or stars. So what do they consist of? To answer this we need to introduce some basic ideas about electricity and magnetism.

You are no doubt familiar with the behaviour of magnets. If a small steel pin is placed close to a magnet, such as the one provided in your Home Experimental Kit, it is pulled towards it, i.e. it experiences a force of attraction.

Do you expect the force of attraction to depend upon the distance between the pin and the magnet?

The closer the pin to the magnet the greater is the force. The magnet is, therefore, exerting some kind of influence that pervades space, the strength of this influence falling off with distance. Thus, corresponding to each position in space relative to the magnet, there is a characteristic force that will be exerted on the pin were the pin placed in that position.

The same kind of thing is found with electricity. Electrical effects are produced by what are called *electric charges*. There are two types. Bodies carrying charges of the same type repel each other whereas bodies with dissimilar charges attract each other. One type of charge is called 'positive', the other 'negative'. The choice of name is quite arbitrary.

Two positive charges repel/attract?
Two negative charges repel/attract?
A positive charge repels/attracts a negative?

Two positive charges repel each other as do two negative charges, and a positive and negative charge attract each other.

All matter is composed of *atoms*. Each atom, as you will see in detail later, has a composite structure. Some of its component parts are tiny particles called *electrons* and these carry a quantity of negative charge. Each electron carries exactly the same amount of charge. In addition to the electrons there is at the centre of the atom a much heavier particle called the *nucleus*. The nucleus carries a positive charge, and this positive charge is exactly equal in magnitude to the sum of the negative charges carried by the electrons.

Do you expect a charged body to experience a force when placed in the vicinity of an atom?

A charged body placed close to an atom normally experiences no net force. The force exerted on it by the electrons is cancelled out by the

equal force exerted by the nucleus in the opposite direction. The atom is therefore regarded as having a net charge of zero—it is said to be 'neutral'.

Atoms are not 'indivisible', as was once thought. Depending upon the type of atom involved, it is more or less easy to remove an electron from it. So, if two dissimilar materials are vigorously rubbed against each other, the one made of atoms with the more loosely bound electrons is likely to lose some of them. They are transferred to the other piece of material. In this way both objects acquire an unbalanced charge—one by virtue of having too many electrons, the other through having too few.

It is then that the electric charges work to produce large-scale effects that are easily visible. These include the way in which strands of hair sometimes repel each other and stand apart when a comb has been passed through them. A lightning flash is another example; this is an electrical discharge initiated by accumulated charges in the atmosphere.

Thus, an electrically charged object exerts an influence throughout space, in much the same way as a magnet does. Once again it is found, when the phenomenon is investigated quantitatively, that the force exerted on another body depends upon the distance to that body, and it progressively diminishes the greater the distance.

In this way, one comes to think in terms of *fields**—electric and magnetic (and later we shall add gravitational and nuclear). An electric charge is thought of as being surrounded by an electric field. This is a 'condition' produced by the charge and this condition pervades the surrounding space. It can be assigned a magnitude and direction at every point. The electric field at a point is specified (a) by the strength of the force that a body carrying a given amount of positive charge would experience if placed at that point, and (b) by the direction in which that force would act. If the magnitude of the force on the body when at one point is twice that when placed at some other point, then the field at the first point is said to be twice that at the second.

This term 'field' is a difficult one to explain satisfactorily. Nonetheless the concept is a most useful one for it allows the analysis of force problems to be divided into two convenient parts: In the first place, one charged body, A, is regarded as being the *source* of the field—the field is there whether or not there are any other bodies to be affected by it—while any other body, B, is thought of as being *acted upon* by the field. The force on B arising from the action of this field can then be studied. Similarly the force on A due to the presence of B can be studied by considering B to be the source of a second field—one that acts on A.

Clearly, while an electric charge remains stationary, the direction and magnitude of the field it produces remains constant at all points.

What do you expect will happen to the field if the charged body is moved to another position?

If, however, the charge is moved to another position the field has to change to one that is appropriately centred around the new position. Suppose a charge is initially situated at position A in Figure 6. The field at the equally spaced points a, b, c, d . . . acts in the directions indicated by the solid arrows, i.e. directly away from A. If the charge is moved to position B, the field has to be directed away from this new position, i.e. acts along the dotted arrows.

* *We are using the word 'field' here in a very special way. Forget other connotations the word has for you, such as ones closely associated with the idea of area, field of view, grass, etc.*

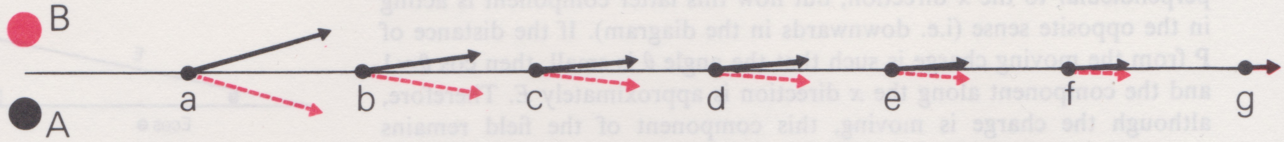


Figure 6

When a charge is situated at position A, the electric field at points a, b, c, d, e, f, g, . . . act along the directions indicated by the solid arrows. When the charge is at position B the field acts in the direction of the dotted arrows. We show the length of the arrows becoming progressively smaller to remind you that the strength of the field falls off with distance. (The rate at which the field strength falls off with distance is in fact much more rapid than is indicated by the lengths of the arrows.)

However, this reorientation of the field does not take place instantaneously. It is found that a certain time, t, has to elapse before the field at point, a, changes. If the distance of the charge to point a is x, and that to point b is $2x$, then it takes time $2t$ before the appropriate change is initiated at point b; $3t$ for point c, etc. In other words the disturbance to the field moves out from the moving charge with a certain velocity given by x/t.

Having moved the charge to the second position, it could immediately be returned to its original position so as to restore the *status quo.*

The charge is back to its original position, but is everything restored to its original state?
Was that last sentence correct?

However, the *status quo* has not been exactly restored. This is because the disturbance produced in the field still carries on out into space. For example in Figure 6, having moved the charge to position B and allowed the disturbance to reach point d, one could return the charge to position A. The initial disturbance to the field is unaltered by this latest movement of the charge and still carries on from d to points e, f, g . . . (with, of course, a second disturbance now following it some way behind corresponding to the movement of the charge from B back to A). Here indeed is a vivid example of the usefulness of the field concept. By only looking at the charge after the movement is completed, one cannot tell that anything has changed. Indeed, as far as the charge is concerned, nothing *has* changed—it is back where it started. A study of the charge alone therefore is inadequate to explain why some time later a second charge at, for example, point g suddenly experiences an abnormal force. Once having introduced the concept of a field that is distinct from either of the two charges, however, one can explain the behaviour of the second charge in terms of a disturbance to the field.

Now, instead of keeping the source stationary after bringing it back to its original position, one can, if so desired, repeat this movement over and over again at a steady rate. Suppose in Figure 7 the charge oscillates between positions A and B. What is the effect of this on the field at P?

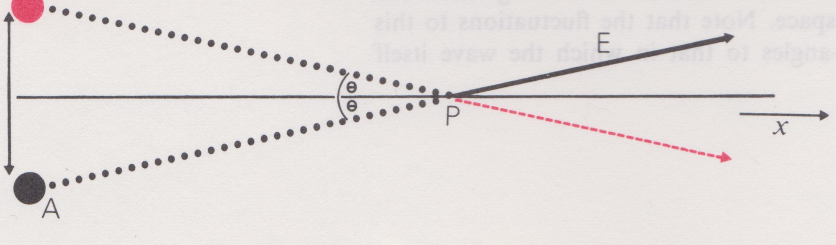


Figure 7

The field at P appropriate to the charge being in position A will have a strength, E, directed along the solid arrow. This can be regarded as being made up of two components $E \cos \theta$ along the x direction and $E \sin \theta$ at right-angles to x (Fig. 8).* When the charge is in position B, the field has the same strength, E, but is directed along the dotted arrow (Fig. 7). This can be resolved once again into $E \cos \theta$ along the x direction and $E \sin \theta$

* *Refer if necessary to* MAFS, *section* 4.D.7.

perpendicular to the x direction, but now this latter component is acting in the opposite sense (i.e. downwards in the diagram). If the distance of P from the moving charge is such that the angle θ is small, then $\cos\theta \approx 1$ and the component along the x direction is approximately E. Therefore, although the charge is moving, this component of the field remains essentially constant. The interesting component is the one perpendicular to x; this is the one relevant to our discussion on the nature of light. It fluctuates in value. (Actually its magnitude exceeds the value $E \sin\theta$ expected on the basis of Figures 7 and 8. This is because the *motion* of the particle generates an additional field in this direction.) Figure 9 shows a graph of how the strength of this field changes with time at a given position.

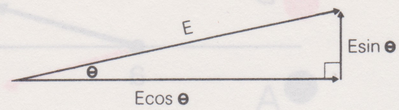


Figure 8

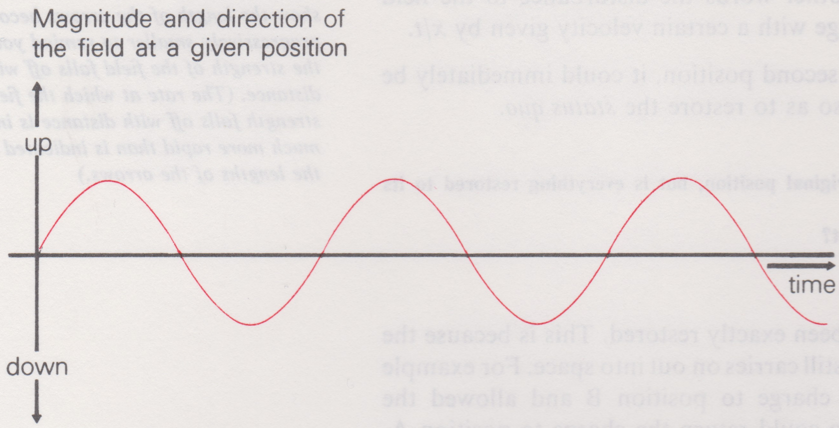


Figure 9

Diagram showing how the fluctuating part of the field at a given point on the x axis *varies with time when the charge oscillates between positions A and B* (see *Figure* 7).

How could we check experimentally that this was the way the field varied?

This can be demonstrated experimentally by placing a small charged body at point P and measuring how the force on it varies with time. If it were free to move at right-angles to the x direction, it would oscillate up and down. In fact it would behave rather like a cork bobbing up and down on the surface of some water over which a wave was passing. Indeed, can this fluctuating part of the field be regarded as a wave of some sort?

To answer this, imagine there to be a number of small charged bodies strung out all along the x direction of Figure 7. At a given instant of time, the forces exerted on each of these bodies by the field are measured. These measurements in turn give the values of the fluctuating field at each position. Figure 10 shows the sort of result one might get.

So in the way the electric field varies, both with time at a given position and with position at a given instant of time, it behaves like a wave train. The quantity that is altering is not of course the position of the particles of a medium, as was the case for sound waves, but the magnitude and direction of the electric field in space. Note that the fluctuations to this field are in a direction at right-angles to that in which the wave itself is moving (the x direction).

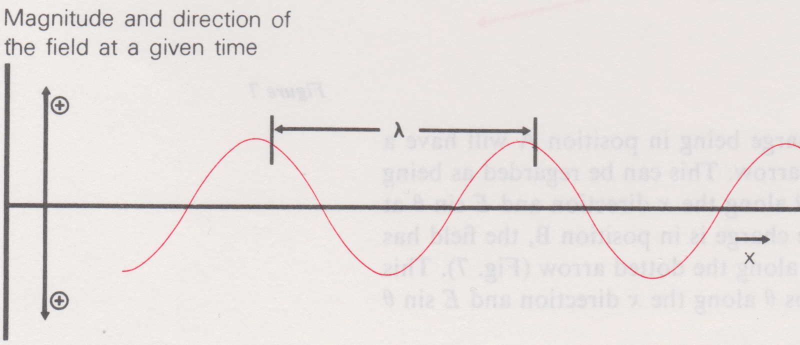


Figure 10

Diagram showing how at a given time the fluctuating part of the field varies from point to point along the x axis when the charge oscillates between positions A and B (see *Figure* 7).

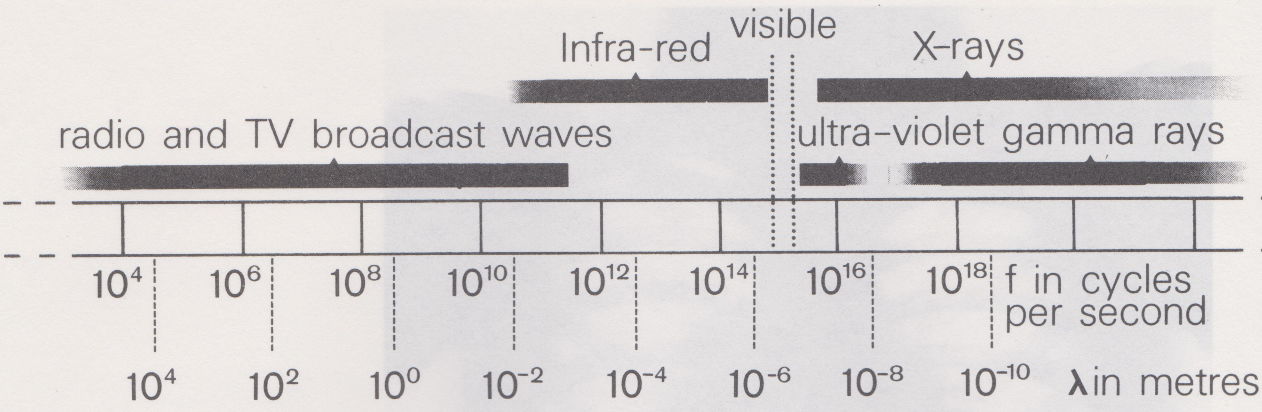


Figure 11

The electromagnetic spectrum.

The number of times per second that the fluctuating component is found to have a maximum value in the same direction at a given point is the same as the number of to-and-fro vibrations of electric charge; it is the frequency of the wave. The wavelength of the wave shown in Figure 10 is then related to this frequency and to the velocity of the wave by the same formula as applied to the sound waves.

Do you remember the formula?
If not turn back and look it up.

If you are still not absolutely happy about this concept of fields, turn to Appendix 3 (Red) (p. 52).

You might find the analogy described there helpful.

As you will see in Unit 4 there is a very intimate relationship between electric and magnetic fields. In fact, whenever an electric charge is moved, a magnetic field is produced. The oscillating charge in Figure 7, therefore, not only produces fluctuations in the electric field but also gives rise to a fluctuating magnetic field. These magnetic disturbances travel out with the electrical ones and with the same velocity and frequency.

Together, the electric and magnetic disturbances are called *electromagnetic waves*. It is these waves, or at least a restricted class of them, that the eye detects. There is no theoretical limitation on the frequencies allowed to electromagnetic waves and the phrase 'electromagnetic spectrum' has been coined to embrace all vibrations of this type. The word 'spectrum' implies that one is distinguishing between various types of wave according to their frequency (or wavelength).

Figure 11 displays the electromagnetic spectrum. As can be seen, it includes a whole variety of phenomena. Just beyond the visible region there are infra-red and ultra-violet radiations.* The former is mainly responsible for the warmth you feel when you place your hand close to a heated electric iron, the latter gives rise to the tanning of your skin on exposure to strong sunlight. X-rays and the waves used in radio and TV broadcasting are familiar enough. For the time being we shall say nothing of gamma rays; they are best described later when we come to the subject of nuclear physics.

2.3.2 The sense of sight extended

The eye responds to only a narrow band of wavelengths. Within this range, light of differing wavelength can be distinguished by colour. There will be a colour plate showing the visible spectrum in Unit 6. It extends from

** The word 'radiation' is sometimes thought of as having special reference to 'atomic radiation'. We, however, shall be using it in the quite general sense of 'something emanating or diverging'.*

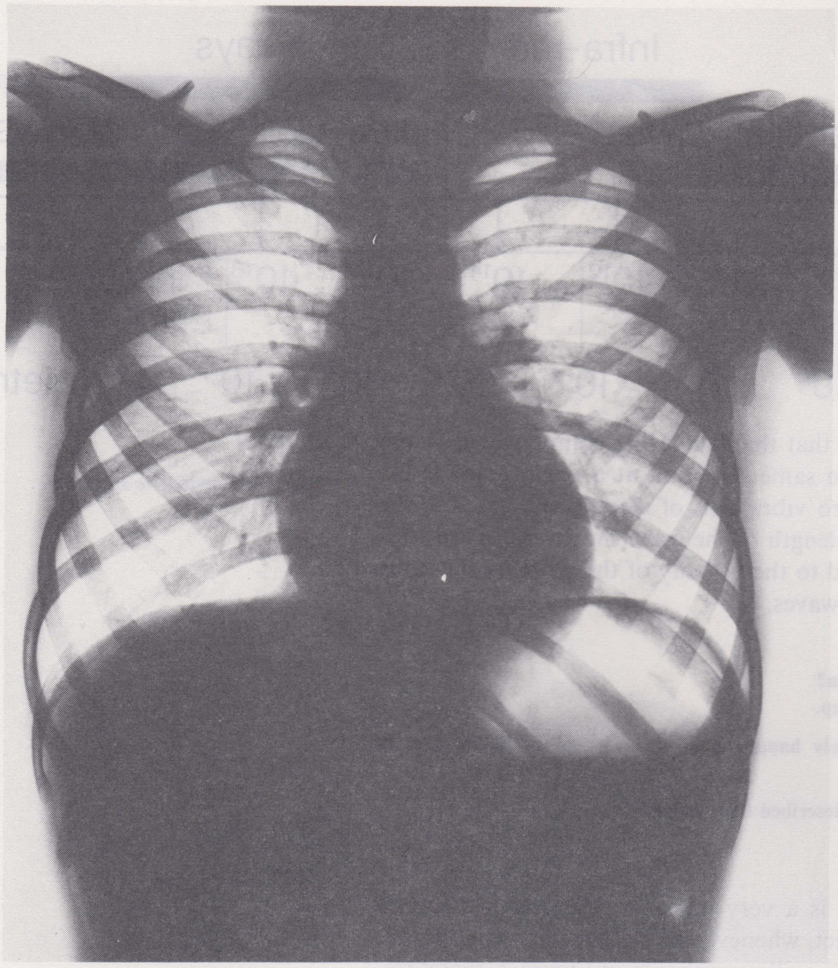


Figure 12
X-ray photograph of a normal healthy person.

violet at one end, corresponding to a wavelength of 400×10^{-9} m, to red at the other, with a wavelength of 700×10^{-9} m. Ordinary white light is made up of a mixture of all these colours. Though the eye can differentiate between light of different wavelengths when they are shown separately, it is not a good analyser of mixtures of light of various wavelengths; it cannot 'see' the component colours of white light, only a single impression of 'whiteness'. In this respect the ear is superior; though an entire orchestra is playing the ear, as you know, can analyse the sound and pick out individual notes. A glance at Figure 11 is sufficient to appreciate how much is missed through the human eye having access to only the visible range.* To the scientist much of the most interesting information about his world is contained in electromagnetic radiation outside the range available to his unaided eye. Thus once again he has to overcome a physical limitation of his body by using auxiliary devices.

There are many such devices, each covering a certain spectrum of wavelengths. One of them has a particularly wide range of application—the photograph.

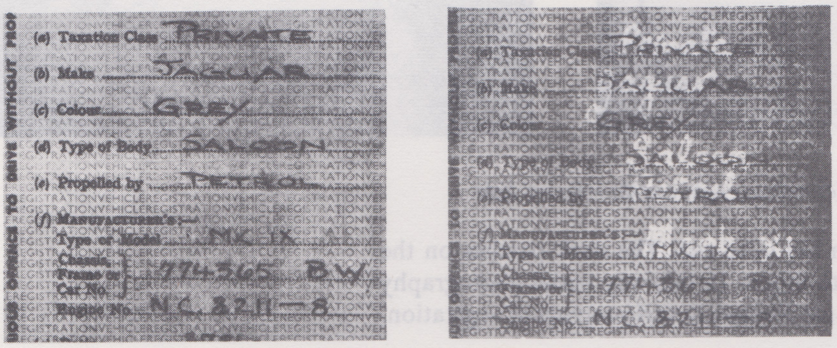
Turn to Appendix 1 for a short description of the principles underlying the photographic technique (p. 47).
Note that this appendix is classed as white-page material and should therefore be read by all students.

The photographic technique can be used to detect radiation of the very smallest wavelengths, right through the visible region and into the infra-red.

** The visible range shown is that appropriate to the human eye and does not necessarily apply to other animals. Bees and moths, for example, can see in the ultra-violet.*

The usefulness of X-ray photography, such as that shown in Figure 12, needs no stressing. As a means of diagnosing disorders within the body, it is of course invaluable. The photograph is that of a normal healthy person, and the position of the heart on the left-hand side of the body is clearly seen.

Photography in the ultra-violet range has been extensively used, for example by police departments, museums and banks, in the detection of fraudulently altered documents, paintings, etc. This use depends upon the relative reflecting powers of paper, inks, erased inks and various pigments. Figure 13 shows a document as seen under (a) ordinary light and (b) ultra-violet light.



(a) (b)

Figure 13

A car log-book viewed in (*a*) *ordinary light,* (*b*) *ultra-violet light.*

On the longer wavelength side of the visible region photographic methods can extend to the infra-red region as far as about 1.0×10^{-6} m. There are many applications for this type of photography. The aerial pictures of Figure 14 show a potato field in the Cambridgeshire fens as seen (a) by ordinary light and (b) by infra-red light. You can see that whereas the appearance of the field is uniform in ordinary light, there is a very marked

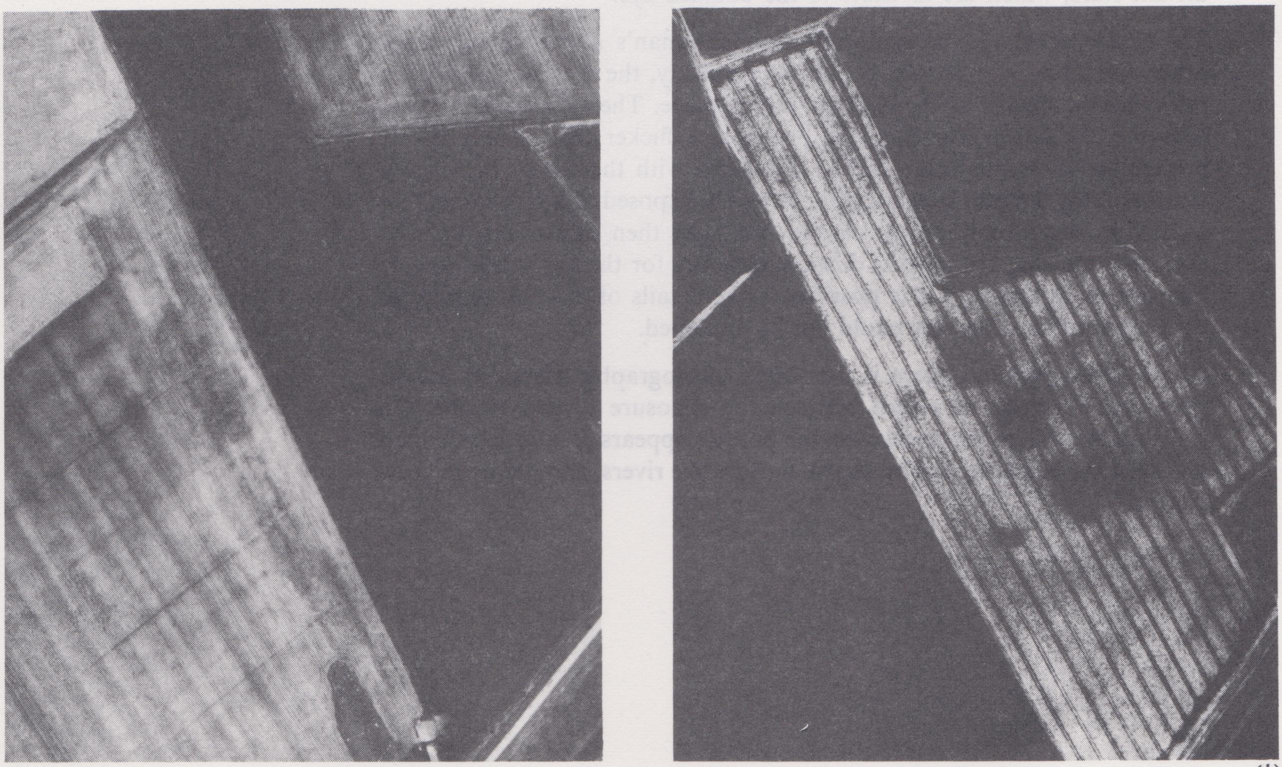


Figure 14 (*a*) (*b*)

A potato field in the Cambridgeshire fens. The dark patches on the infra-red photograph on the right reveal the presence of potato blight not visible on the panchromatic film on the left.

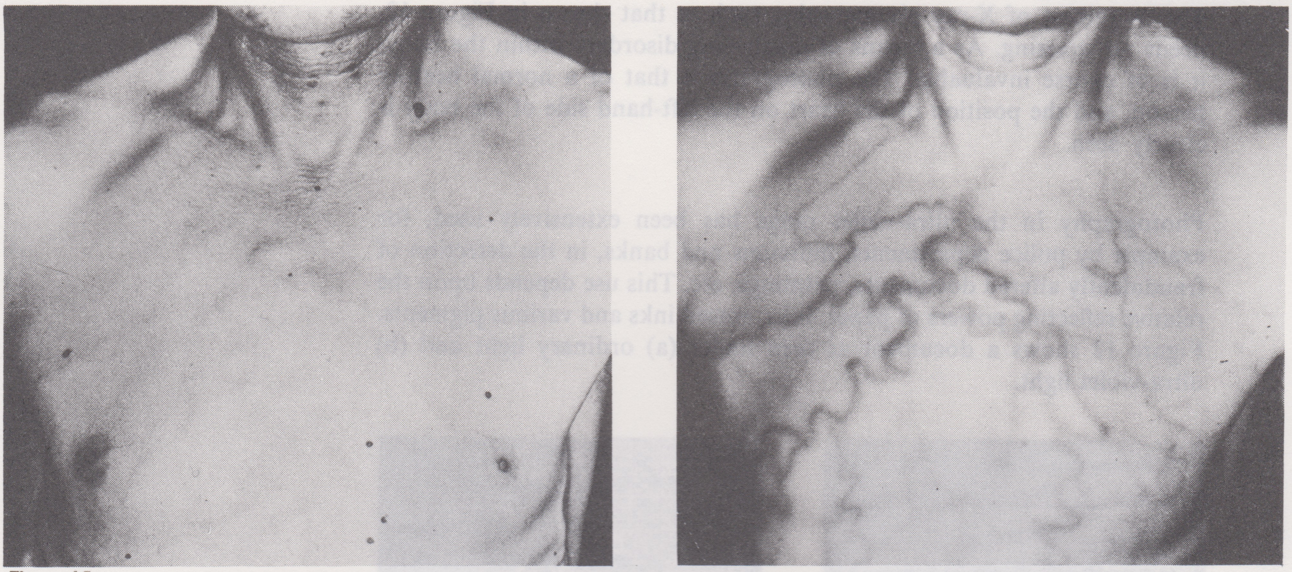


Figure 15

Photographs of a man's chest taken in (a) ordinary light and (b) infra-red light.

difference when it is looked at in the infra-red. The dark patches on the latter reveal the presence of potato blight. In this way aerial photography enables one rapidly to diagnose some diseases in various types of vegetation.

In the medical field infra-red photography can be used in the examination of veins (Fig. 15). Another application is to situations where it is important that the subject remains in 'darkness', for example when one wishes to study the night behaviour of warmblooded animals (or indeed of burglars).

Quite apart from this ability to extend the sense of sight into wavelength regions not accessible to the eye, the photographic technique can be of great use in other respects. For example, the emulsion can be exposed for long periods and thus be made to reveal very faint sources, such as distant stars, which are invisible to the unaided eye.

The development of cinematography extends man's ability to see in yet other directions. If a process occurs very quickly, the eye has difficulty in resolving the details of the changes taking place. The spot of light flying over the TV screen, for example, produces a flicker that is only barely perceptible. This limitation can be overcome with the use of high speed photography. With this technique, frames are exposed at a rate sometimes as high as several million per second. They are then shown at a reduced rate, thus slowing the process down sufficiently for the eye to see what is happening. In this way it is possible to see details of a whole variety of phenomena that otherwise could not be discerned.

At the other extreme, there is time-lapse photography. Here one allows long time intervals to elapse between the exposure of each frame. On showing the film at the usual rate, the process appears greatly speeded up. Glaciers, for example, can be shown to flow like rivers, and plants to grow visibly.

2.4.1 The Need for Quantitative Measurement

You have now seen how man has been able to extend his senses. Unfortunately, his senses are easily misled; they do not always provide reliable information. His sensory receptors can become fatigued or 'adapted' by prolonged stimulation. Here we remind you of a few well-known examples.

You are no doubt already familiar with visual after-images. If you stare fixedly at the cross in Figure 16 for about a minute, and then transfer your gaze to a plain white background, a bright after-image of the cross will be seen. Try it.

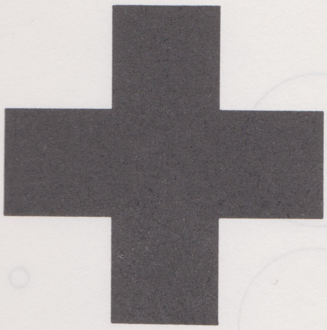


Figure 16

If you carry a heavy suitcase for a long period, on putting it down your arm suddenly feels light.

The sense of taste can be conditioned. The sweetness of a cup of coffee is judged differently depending upon what one is eating at the time—a piece of cake perhaps or something savoury.

We leave it to you to decide whether this is an opportune moment to try this one!

The impression of time 'dragging' or 'flying' depends upon the activity in which one is engaged. Prolonged driving at 70 mph on a motorway can alter one's view of the reasonableness of a subsequent 30 mph speed limit in a built-up area.

Judgement of temperature is subjective. This can be demonstrated by holding your left hand in cold water for a few minutes and your right hand in hot, and then plunging both simultaneously into a bowl of lukewarm water. Try it.

What conclusions do you draw about the hotness or coldness of the water in the bowl based on the reaction of each hand?

Some misleading impressions are due to modifications made to the behaviour of the sense receptors. In response to identical stimulation, receptors transmit different nerve impulses depending upon the level of prior stimulation. Thus, in the case of the after-image effect, prolonged exposure to light reduces the sensitivity of the receptors in the eyes. When the eyes are subsequently directed to a uniformly bright surface, the receptors previously exposed to light transmit fewer nerve pulses than those that were not similarly exposed. The way in which the nervous system operates will be dealt with more fully in Unit 18.

However, not all observational distortions can be attributed to such changes. For example, you will already be familiar with optical illusions such as those shown in Figure 17.

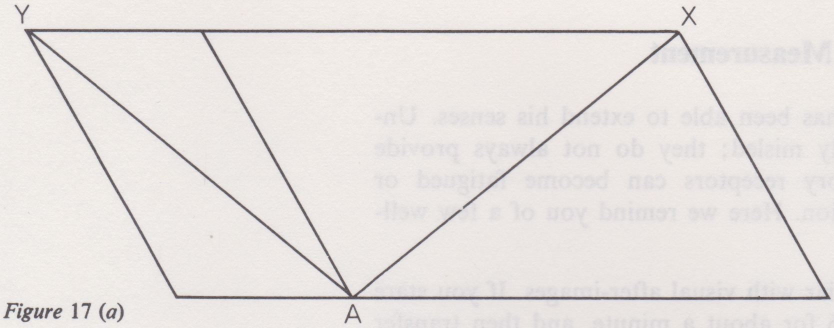


Figure 17 (*a*)

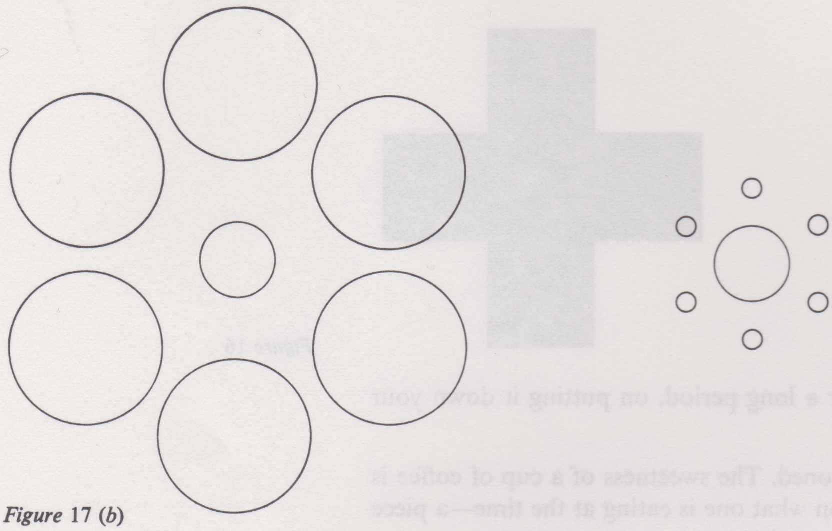


Figure 17 (*b*)

Ebbinghams' figure.

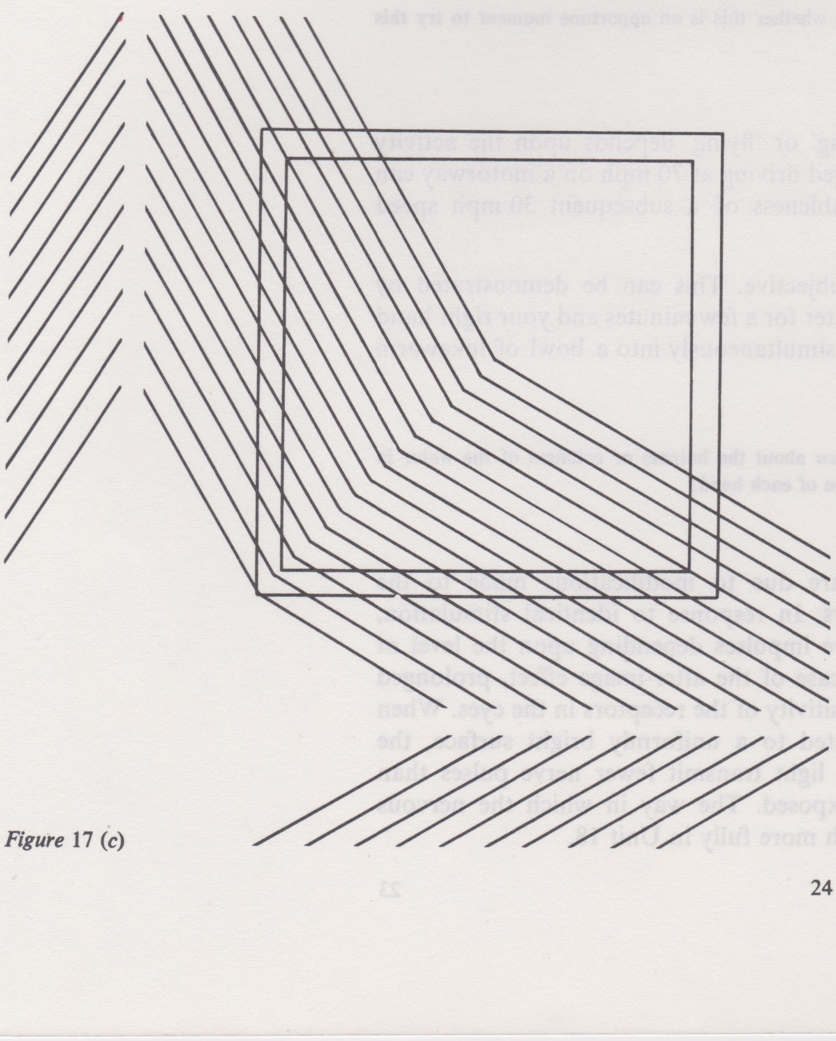


Figure 17 (*c*)

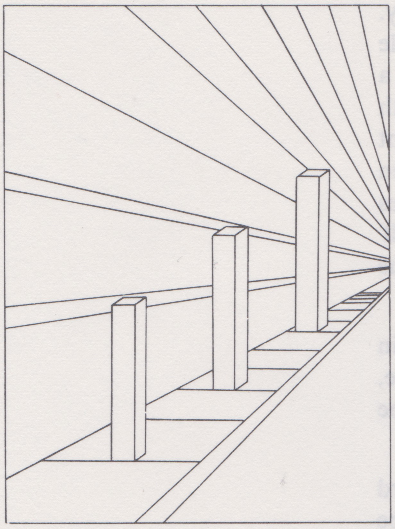


Figure 17 (*d*)

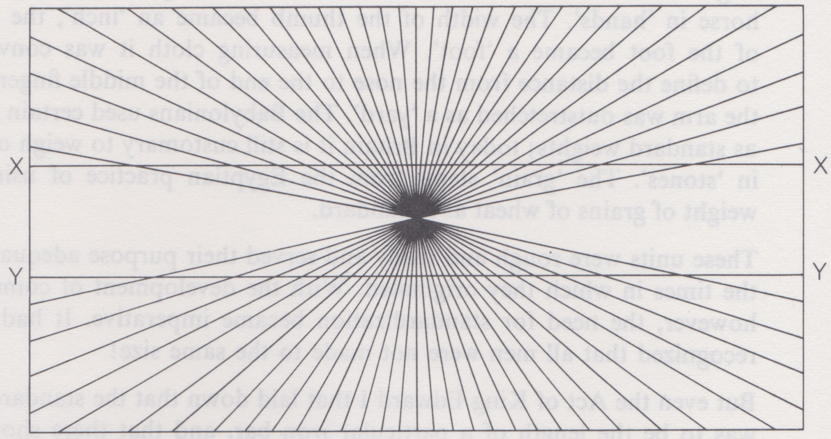


Figure 17 (*e*)

Hering's illusion.

17 (a) Are lines XA and YA of the same length?
17 (b) Are the inner circles identical?
17 (c) Is the figure a square?
17 (d) Are the upright columns identical?
17 (e) Are the lines XX′ and YY′ straight and parallel?

It is fairly well established that these latter types of illusion originate, at least primarily, in the way in which the brain organizes the information sent to it. Experiments have been performed in which the subject was able to see only part of a drawing with one eye, the rest being seen by the other eye. Under these circumstances the illusion remains. As neither eye sees the whole drawing the illusion cannot be attributed to the functioning of the eye but instead to the way in which the brain fuses together the information.

Clearly observations have to be interpreted with caution. Indeed variations in observation become even more marked when one person's experience is compared with that of another. Individuals see colours differently—this is known because some people are colour blind. Individuals hear sounds differently—some are tone deaf.

If one is to add to a common fund of knowledge information that is to be useful to others, it must be about objective reproducible experience; it must be free from the whims and peculiarities of the particular observer. Observation must develop therefore into something more quantitative —it must become *measurement*.

2.4.2 Systems of units

In measurement, certain standard *units* are agreed upon and all other quantities are expressed as multiples of these units. For example the length of a particular object, say a metal rod, may be chosen as the unit of length. The lengths of all other objects may then be related to the length of this rod. An object may be twice as long as the rod (i.e. of length two units), three-and-a-half times as long (3.5 units), a quarter as long (0.25 units), etc. Once the unit is established, measurement becomes a matter of counting.

The question arises as to what basis one should choose for the system of units. The choice is, of course, an entirely arbitrary one. Not surprisingly, therefore, many systems have appeared at different times.

Early units of length tended to be related to the size of the human body. A 'cubit' was the distance between the elbow and the end of the middle finger. The width of a man's hand was used to express the height of a horse in 'hands'. The width of the thumb became an 'inch', the length of the foot became a 'foot'. When measuring cloth it was convenient to define the distance from the nose to the end of the middle finger when the arm was outstretched as a 'yard'. The Babylonians used certain stones as standard weights; today in Britain it is still customary to weigh oneself in 'stones'. The 'grain' stems from the Egyptian practice of using the weight of grains of wheat as a standard.

These units were rough and ready and served their purpose adequately in the times in which they originated. With the development of commerce, however, the need for standardization became imperative. It had to be recognized that all men were not made to the same size!

But even the Act of King Edward I that laid down that the standard yard was to be the length of a particular iron bar, and that there should be exactly three feet and exactly 36 inches in this standard yard, did not entirely alleviate the problem. Other systems of units also became standardized. This multiplicity of systems has left a legacy of confusion—not to mention the considerable and quite unnecessary labour of converting measurements from one system to another.

Can you think of any examples of this kind of confusion?

The problem of having to convert temperatures from degrees Fahrenheit to degrees Celsius (or Centigrade), or a Continental speed limit from kilometres per hour to miles per hour, are just two examples of the trouble caused.

Fortunately, vigorous steps are now being taken to achieve uniformity. In 1960 it was formally approved by the General Conference of Weights and Measures that a system of units called Système International d'Unités (abbreviated to SI units) should be adopted. The Open University publications will in general conform to this decision.

In section 3 of *HED*, there is some further information about units. You will find it useful to refer to this at various stages during the Foundation Course and indeed throughout your degree course, if you continue to study science with us. *Don't* try to memorize it. You will be gradually introduced to the terms as you go along (some of them you will not need at all during the Foundation Year—they are there merely for the sake of completeness).

For the time being we shall concentrate on just two of these units—the *metre* as the unit of length and the *second* as the unit of time.

The unit of length: The metre was originally defined as one ten-millionth of the distance from the Pole to the Equator along a great circle† passing through Dunkirk in France and a point close to Barcelona (Fig. 18).*

This was chosen because it was a distance that could be remeasured at will (and could not be lost!). However, the measurement of it is not at all convenient. So later, two marks one metre apart were inscribed on a metal bar supported in a specified manner, at standard atmospheric pressure†, and at the temperature of melting ice.** This distance was then defined as the new standard for the metre. The bar was kept in a vault at Sèvres near Paris and copies of it were made for other countries.

* *As you can read in* The Roots of Present-Day Science (RPS), *this definition had its origins in the burst of scientific activity that stemmed from the French Revolution.*

** *The temperature must be specified because materials change their dimensions with change of temperature.*

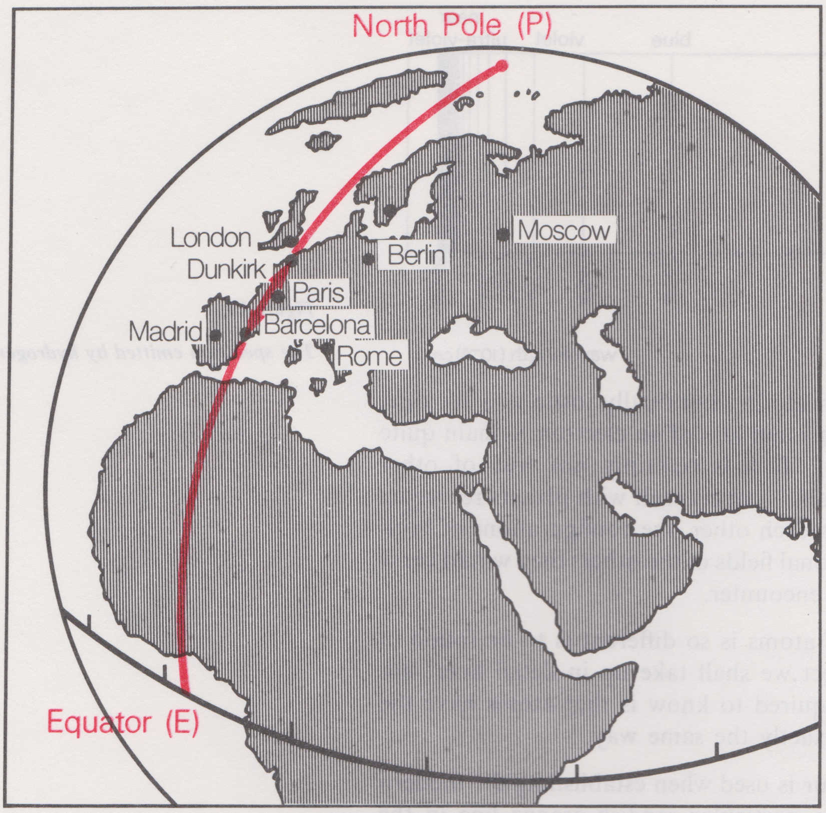


Figure 18

The metre was originally defined as one ten-millionth of the distance PE.

This was still not considered entirely satisfactory—one question that arose for example concerned the exactness of the copies. It was recognized that a still firmer basis for the definition of the metre would be desirable.

The search was for an ideal standard—one based on a property of nature that could be relied upon never to change and was moreover readily accessible to everyone without the need to make copies. It is at this point we introduce you to a remarkable feature of nature—the identical behaviour of atoms.

We said earlier that there were different types of atom; each is characterized by the number of electrons it contains. Substances built up from atoms of one type are called *elements*. Silver and bromine are two elements you came across in Appendix 1 on photography. Atoms of a given element are identical*; they have identical nuclei and identical electrons. There are a variety of possible configurations the electrons can adopt within the atom, but these possible configurations are the same for all atoms of the same element.

An electron can move from one allowed configuration to another, i.e. alter its orientation with respect to the nucleus. When making certain transitions it emits a pulse of light. The wavelength of this light is a characteristic of the transition from a particular initial configuration to a particular final configuration. These pulses of light are of extreme importance because they carry much information about what goes on inside the atom. As you will see in some later Units, by measuring the wavelength of the light emitted, one can infer which particular transition the electron has executed. Figure 19 shows a spectrum of the light emitted by atoms of the element hydrogen. In contrast to the continuous white light spectrum you will be shown in Unit 6, here you see that only certain wavelengths are present. It is called a *line spectrum*. This set of lines is characteristic of hydrogen; no matter where or when it is measured, no matter what the past history of the particular sample of hydrogen may be, this is the spectrum obtained.

* *Not strictly true—there are varieties called isotopes, but these are irrelevant to our discussion. They will be dealt with in Unit 6.*

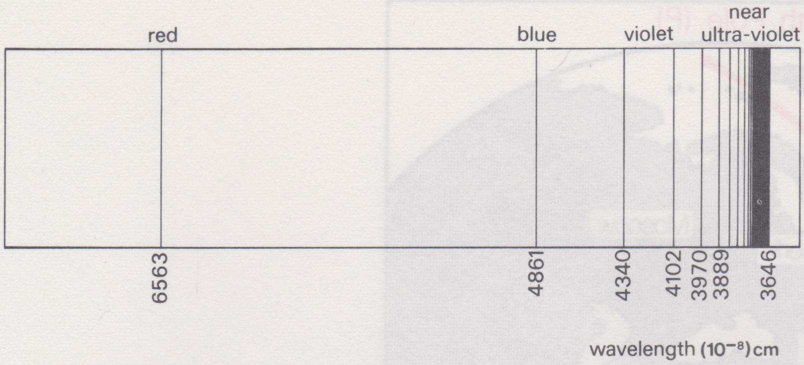


Figure 19

The spectrum emitted by hydrogen.

Why should this be so? How can atoms continually jostle and hit each other and yet, apart from the occasional loss of an electron, remain quite unchanged by their past history? This is certainly not true of other systems one can think of. If two solar systems each with planets revolving around a central sun came near to each other, the configurations of both would be distorted by the gravitational fields of the other; they would carry away with them the effects of the encounter.

The reason why the behaviour of atoms is so different is to be found in Quantum Theory. This is a subject we shall take up in detail later, but for the time being all you are required to know is that atoms have the property of always behaving in exactly the same way.

This exactly reproducible behaviour is used when establishing the modern standard of the metre. In 1960, a particular reddish-orange line in the spectrum of the gas krypton was chosen, and *the standard metre is now defined as having a length equal to 1 650 763·73 wavelengths of this light in a vacuum.** This may seem to you an extraordinarily odd definition; but in practice it is very convenient. Krypton is readily available all over the world because it is present to a small extent in the atmosphere. So anyone at anytime has now direct access to the standard—he has no need to refer to a particular object kept in a vault somewhere.

The unit of time: We turn now to the definition of a unit of time. Here, unlike length, it was not possible to take a certain quantity, define it as the standard and keep it locked away in safety. It was necessary to think of something else.

The notion of the passage of time is intimately bound up with the repeating cycle of day following night. So it first appeared natural that the standard of time should be related to the length of a day. It could be given as a fraction of the solar day (i.e. a fraction of the period of time from the moment when the sun is at its highest elevation in the sky one day, to the corresponding moment the next day). However, the length of the solar day varies throughout the year. So the unit must be based on the *mean solar day*, which is an average taken over the whole year. The division of this mean solar day into 24 hours, each hour into 60 minutes, and each minute into 60 seconds, gives the basic unit of time—*the second.*

Unfortunately the rotation of the Earth is not exactly regular—for one thing the action of the tides gradually slows down the rate at which the Earth spins on its axis, and so lengthens the mean solar day. A more consistent type of periodic behaviour is therefore required.

* *We must specify 'in a vacuum' because although a light wave does not* need *a medium in which to travel, if it* does *pass through a medium (such as the air) then it slows down. The frequency remains unaltered so we see from the relation* $v = \lambda f$ *that the wavelength must be affected if the velocity is lower. Note you are* not *required to remember the number of wavelengths.*

Where do you suggest one could look?

Once again the ever-constant properties of the atom can be used. The velocity of light in a vacuum is the same for light of all wavelengths, so the characteristic wavelength of a spectral line specifies through the equation $v = \lambda f$ a frequency which is also characteristic of the line. This reproducible frequency can then be used to fix a standard of time. The basic unit of time can be defined as the period needed to complete a certain number of cycles at that frequency. Thus the constant behaviour of atoms makes it possible to establish standards of both length *and* time.

Since 1967 a line in the spectrum of the element caesium has provided the basis of the unit of time. *The standard second is defined as that time interval occupied by 9 192 631 770 cycles of this radiation.** A standard clock using atoms of caesium as its effective 'pendulum' can be made to keep time to an accuracy of better than one second in 10 000 years.**

So you have now reached a point where you understand how two of the basic units listed in *HED*, section 4, originated.

You might take this opportunity of memorizing *HED*, section 3.1.5 on 'Fractions and Multiples'. We can then start to use the prefixes. Also note in section 3.1.3 that the unit for frequency is the *hertz*,* abbreviated to Hz. From now onwards we shall drop the use of 'cycles per second', which means the same thing.**

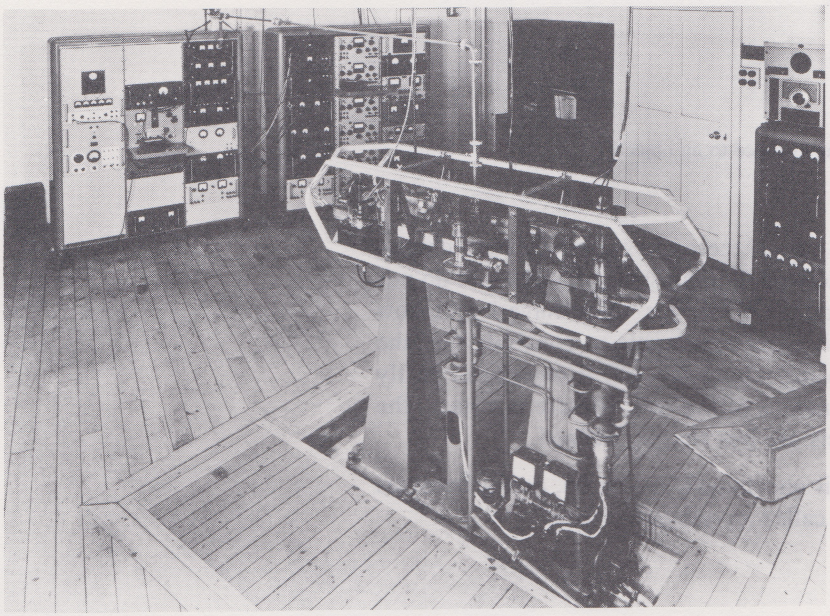


Figure 20

The world's first Caesium clock developed at the National Physical Laboratory, Teddington.

* *You are not required to remember this number.*

** *Before ordering one for the home, however, you are advised to look at Figure 20—it's not exactly what one normally expects a clock to look like.*

*** *Named after the German experimental physicist, Heinrich Hertz, who did much work on the nature of electromagnetic radiation.*

Section 5

2.5.1 The Need to Extend the Scale of Measurement

You will have noticed that the early units of length were closely related to the size of a man's body; the unit of time to the length of a day. Such choices are not surprising. The units were devised when man was most preoccupied by his immediate observations.

But you have already seen that there is no reason why man should restrict his observations to what is directly accessible to his senses. He could also with a little ingenuity extend the scale of his measurements. But why should he want to do this? The reason is simple enough. Many of the problems that excite his curiosity or whose solution would be technologically most advantageous can only be answered by measurements made over extreme scales of length and time.

What is man made of? The answer is partly to be found in the study of tiny atomic particles. This is a world where movement is rapid and changes occur quickly—it involves, therefore, not only very small distances but also short time intervals.

How and from what did he evolve? To answer this he must find means of measuring time that stretches far beyond the span of his own existence on Earth.

These and other questions you will be tackling in this Foundation Course.

2.5.2 Measurement of large distances

How do you normally judge the distance to an object?

When judging the distance to an object, there are several clues to choose from. If your eye registers a minutely small impression of a bus, you do not immediately think 'there is a minutely small bus'. You have learnt how big a bus is normally, and so it is more reasonable to conclude that the apparent smallness is due to its being a long way off. If one object partly obscures another, it must be the nearer of the two. If there is a haze, the colours of the more distant objects are less sharply defined.

With *two* eyes, depth perception is enormously increased, and it becomes possible 'to see in depth'. This is called *stereoscopic vision.*

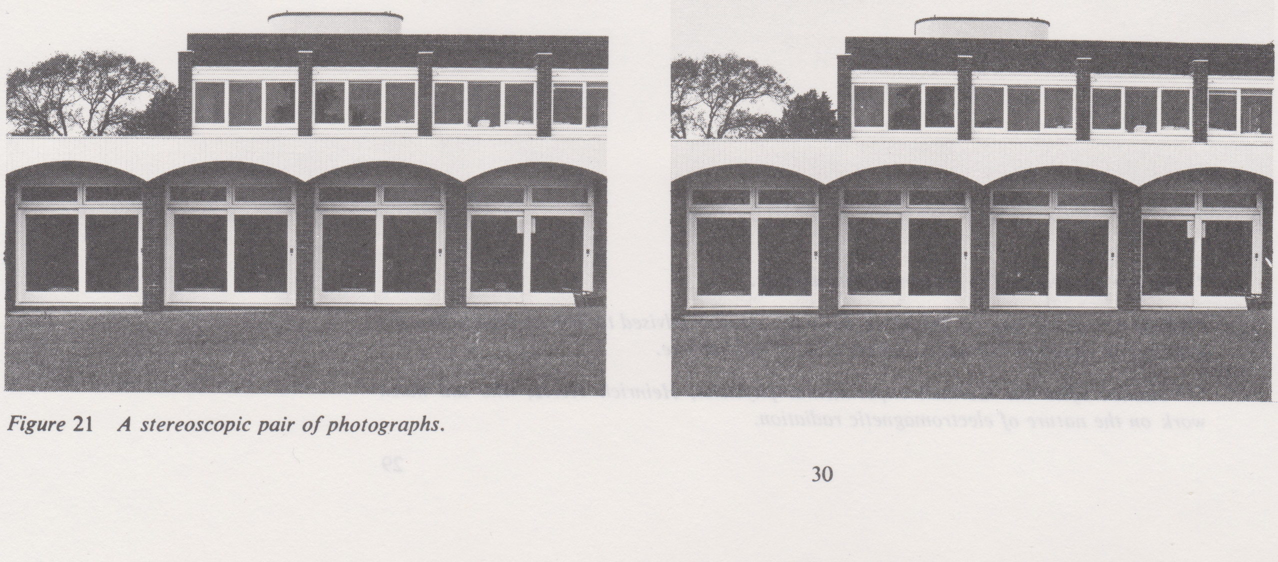


Figure 21 *A stereoscopic pair of photographs.*

The principle of stereoscopic vision is easy enough to understand. The two eyes are spatially separated, so they look at an object from slightly different vantage points. Figure 21 shows views of the library building of the Open University at Milton Keynes as seen from two such points. The relative positions of the upper and lower storey appear to have shifted, the amount of shift depending upon their distance from the camera. It is still a problem, of course, as to how the brain manages to put two pictures together to give an immediately recognizable impression of varying depths. But at least the information on relative depths is known to be there on the pictures. If presented with two pictures taken by cameras separated by a known distance, it would be possible to calculate the distances to the objects.

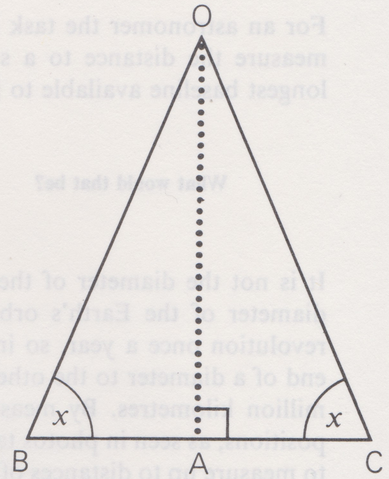


Figure 22

In Figure 22 the positions of two cameras are shown as points B and C. Directly ahead of them is an object at O.

From simple trigonometry*

$$\frac{OA}{AC} = \tan x$$

$$\text{i.e. } OA = \frac{BC}{2} \tan x \quad \text{.........(2)}$$

Suppose the cameras are situated 12 metres apart and the angle, x, is 84° for an object placed at point O in Figure 22. What is the distance to the object from the centre of the baseline?

$BC = 12$ metres $\therefore \frac{BC}{2} = 6$ metres

$\tan x = \tan 84° = 9.514$ (from a table giving the values of tangents of angles)

$\therefore$ From equation (2) the required distance, OA, is given by

$OA = 6 \tan 84° = 6 \times 9.514 = 57.084$

$OA = 57$ metres

So for each value of the distance OA there is a characteristic value of the angle x, or conversely if one knows the angle x and the distance BC, the distance OA can be deduced.

The distance BC is called the *baseline* and this method of estimating distance is known as *triangulation*.

We have chosen a particularly simple configuration in which the object was dead ahead, i.e. $\angle OBC = \angle OCB = \angle x$. When this condition is not satisfied, the mathematics becomes a little more complicated but the principle remains the same—given the angles and the baseline, the distance to the object is uniquely determined.

If you look at a table of values for the tangent of an angle you will see that, if the angle is close to 90°, a tiny change in angle makes a big difference in the value of the tangent.

In what way does this affect our estimation of distance?

If angle x in equation 2 is close to 90°, a slight error in its determination introduces large inaccuracies in the value of OA. As the distance of the object to the baseline increases this angle approaches 90°. Any particular camera set-up, therefore, is suitable for estimating distances over a certain restricted range and not beyond. The same applies to your two eyes, they are also effective only over a restricted range.

It would be a different matter if the baseline could be extended. You can see from equation 2 that, for a given angle x, the distance OA is proportional to BC. If one could for example double the baseline, one could double the range of the distance measurements.

Extending the baseline is in fact what a surveyor does. He views a distant object with his telescope, first from one position and then from another, the separation of the two positions being known. He can, of course, use very large baselines.

* *See* MAFS, *section* 4.A.1, *if necessary.*

For an astronomer the task is that much greater. Suppose he wanted to measure the distance to a star he would have to look around for the longest baseline available to him.

What would that be?

It is not the diameter of the Earth, as you might think at first, but the diameter of the Earth's orbit around the Sun. The Earth completes a revolution once a year, so in a period of six months it moves from one end of a diameter to the other (Fig. 23). The length of this baseline is 300 million kilometres. By measuring how much stars appear to shift their positions, as seen in photos taken from these two vantage points, he is able to measure up to distances of about 5×10^{15} km. Triangulation is therefore an effective means of distance estimation for the astronomer—it essentially increases the distance between his eyes.

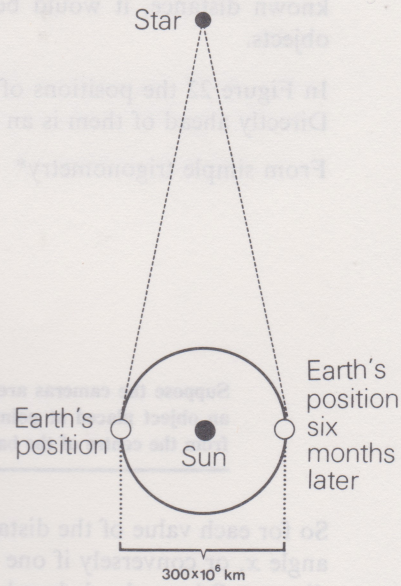


Figure 23

Triangulation using the diameter of the Earth's orbit as base line.

2.5.3 Measurement of small distances

It is obviously not possible to go on subdividing a length indefinitely without reaching a stage where it is no longer possible to see the sub-division. The object could of course be brought closer to the eyes so that it looked larger, but as you know there is a limit to how near an object can be brought for it still to look clear and sharp. So instruments must be used to aid the eye. The simplest is the *magnifying glass* and more complicated one—*the microscope*.

At this point we check your knowledge of simple optics. Do you need to study either or both of the following Appendices?

Appendix 4 (Red) on Lenses:

1. Do you understand the following terms:
- **(i) a ray?**
- **(ii) refraction?**
- **(iii) an image?**
- **(iv) a lens?**
- **(v) lens aberration?**

2. When a ray of light passes through a slab of glass, the angle through which it is deviated depends on four factors. Can you name all four?

If you fail to answer any item study Appendix 4.

Appendix 5 (Red) on The Optics of the Eye.

1. Can you explain the formation of an inverted image on the retina?

2. Can you explain how our eyes are able to focus on objects at different distances?

If you fail on either question study Appendix 5.

NOW (WHETHER OR NOT YOU HAVE WORKED THROUGH APPENDICES 4 and 5) TURN TO APPENDIX 2 ENTITLED 'THE PRINCIPLE OF THE MICROSCOPE' AND READ SECTION 2. THIS IS WHITE-PAGE MATERIAL.

A great deal of expertise goes into the design and manufacture of microscopes in order to produce highly magnified, sharply focused images. But eventually optical instrument makers come up against a barrier—one imposed by the nature of light itself.

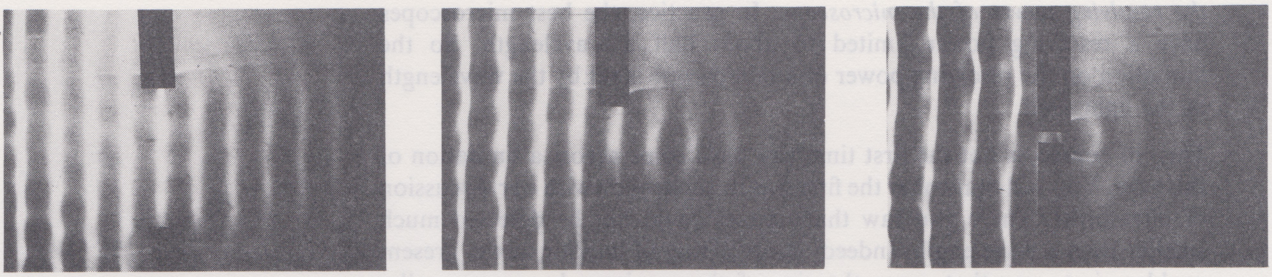


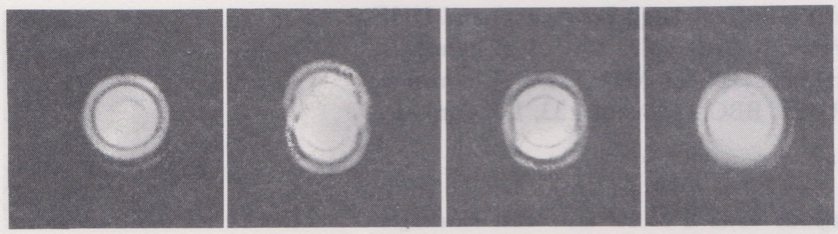
Figure 24

Photographs of water waves passing through a gap in a barrier. Diffraction becomes more apparent the smaller the gap compared to the size of the wavelength.

When discussing the optics of the microscope (in Appendix 2), we drew straight lines in the diagrams to denote light rays. Light however is not so well-behaved. You saw earlier that it had a wave nature. Waves, when they pass close to obstacles, can change direction. To illustrate this, we show in Figure 24 water waves of various wavelengths passing through a gap in a barrier. The waves are travelling from left to right, so this is the initial direction of the rays (you are to imagine them at right-angles to the wave-fronts). When the wavelength is small compared with the size of the hole, the waves passing through it continue mainly in this direction. But when the wavelength of the wave is about equal to, or greater than, the size of the hole, you can see that the wave bends round the corner to a considerable extent—it spreads into regions on either side that you would perhaps expect to be in 'shadow': this characteristic behaviour of waves is called *diffraction*.

We said earlier (p. 14) that an *object* comparable in size to that of a wavelength, scatters only a *little* of the incident wave—most of it continues on undisturbed. Now we are saying that a *hole* comparable in size to a wavelength produces a *large* disturbance with much of the wave being deviated. How do you reconcile these two observations?

There is no contradiction. One would expect an object to cast a shadow. But if the object is small (as in Fig. 5) the waves diffracted round its edges spread into the region behind the object to such an extent that the shadow is almost non-existent. If the intensity of the on-going waves in this region is so little reduced, then clearly not much of the wave is left to be scattered into other directions.



(*a*) (*b*) (*c*) (*d*)

Figure 25

Diffraction patterns produced by a lens: (a) image of a single point object; (b), (c) and (d) images of two point objects being brought closer together.

When light waves pass through a lens set in a microscope, they too are passing through an aperture. The rim of the lens is acting as the boundary of a hole—any light not striking within the rim of this hole is prevented from passing through to the other side by the opaque material of which the microscope is constructed. The light will, therefore, be slightly diffracted (only slightly because the diameter of the hole is very much larger than a wavelength). The expanse of glass straddling the hole serves to concentrate the rays; but diffraction spreads the light out a little and so gives rise to a blurred image rather than a point image. Figure 25 (a) shows the sort of effect produced. There is a central bright region surrounded by alternating rings of light and dark (why the image takes on this particular form will be explained later in Unit 28). Figure 25 (b) shows images of two well separated objects showing similar patterns. In Figures 25 (c) and (d) they are brought closer together. In the latter case it is more or less impossible to decide whether there really is a second object. Thus, if the distance between two objects is smaller than some minimum value, it is no longer possible to resolve them. *This minimum separation is known as*

the resolving power of the microscope. In practice, the best microscopes have a resolving power limited to about half a wavelength. So the limitation on the resolving power of a microscope is set by the wavelength of the light used.

This, of course, is not the first time you have come across a limitation of this type. You will remember the first was in connection with our discussion of ultrasonics. There you saw that objects could not be detected much smaller than a wavelength. Indeed, another way of looking at the present problem is to say that, once the size of the specimen becomes smaller than the wavelength of the light that is illuminating it, it becomes very inefficient at reflecting light into the microscope.

One attempt to remedy this, of course, would be to increase the size of the objective lens. This would increase the angle, ϕ (phi), subtended† by the lens at the object (Fig. 26), and would allow the lens to collect a greater fraction of the light scattered by the object. But the improvement to be gained this way is limited.

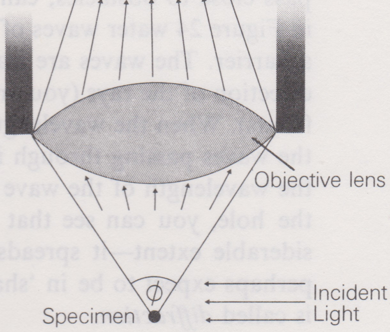


Figure 26

Diagram illustrating the angle subtended by the objective lens at the specimen being viewed under a microscope.

What else could you suggest might lead to an improvement in resolving power?

Clearly an alternative approach is that which was adopted in the case of sound waves, i.e. to reduce the wavelength of the illumination. Does electromagnetic radiation reflect from objects more efficiently if the wavelength is smaller? The answer is—yes. The blue colour of the sky and the redness of sunsets are examples of everyday phenomena produced by this behaviour of light.

At this stage you should do the home experiment (Unit 2, Experiment 1) which goes into this topic further.

A further example of the dependence of the scattering of electromagnetic radiation on wavelength is to be found in the reception of TV waves. In this country we transmit on both VHF (Very High Frequency) and UHF (Ultra High Frequency). On VHF, for example, BBC 1 (Channel 1, London) is transmitted at a frequency of 45 MHz, ITV (Channel 9, London) on 195 MHz; and on UHF, BBC 2 (Channel 33, London) at a frequency of 573 MHz.

If the velocity of electromagnetic waves is 3×10^8 ms^{-1} what wavelengths would these three waves have?

For the first broadcast channel, $f = 45$ MHz $= 45\times10^6$ s^{-1}. From the equation $v = \lambda f$:

$$3\times10^8 = \lambda\times45\times10^6 \quad \therefore \lambda = \frac{3\times10^8}{45\times10^6} = \frac{300}{45} = 6.67 \text{ m}$$

Similarly for the other channels the wavelengths can be found to be 1.54 m and 0.523 m.

The difference in these wavelengths makes it easier to scatter UHF waves compared to the others and so for these there tend to be more 'shadows', or pockets of bad reception.

If you have a conveniently placed indoor TV aerial you might like to get a feel for how easy or difficult it is to shield the aerial from the waves. Try holding or suspending objects of various sizes (preferably metal ones like saucepans, aluminium foil, a dustbin lid, etc.) close to the aerial in a variety of positions. Watch their effect on the picture. Try it for UHF and VHF. (Note that if you are not careful the shielding effect produced by your hand, arm, etc., may swamp the effect you are looking for.)

Thus, in order to see still smaller objects under a microscope it would appear that the answer is to go to smaller wavelengths. Consequently some microscopes use electromagnetic radiation with wavelengths just beyond the visible region, namely ultra-violet light.

What could you use to view the image?

Such microscopes, fitted with cameras as detectors, have proved quite useful.

At still shorter wavelengths there are X-ray microscopes. But here one encounters serious difficulties. In the first place many substances are transparent to X-rays and so scatter little radiation. Secondly, as mentioned in Appendix 4, the refraction undergone by an electromagnetic wave is dependent upon wavelength; for X-rays the refracting effect is very small indeed and so they prove difficult to focus. Some microscopes have been developed which dispense with lenses and focus the rays instead with mirrors made of curved crystals. Even so, the practical difficulties of getting images of good intensity and contrast restrict the performance of the instrument to well below the theoretical limit based on wavelength considerations alone. In fact their ability to resolve small distances is not greatly different from that of microscopes using light in the visible region.

Thus the possibilities of microscopes using electromagnetic radiation as their source of illumination appear to be exhausted. In order to extend still further the measurement of small distances, a radically different technique is called for. We now describe the electron-microscope—a technique that 'illuminates' the object with a beam of electrons.

So far you have learnt that electrons are tiny, negatively charged particles and that they are one of the constituents of atoms. Basically the idea behind the electron-microscope is that a beam of electrons is directed at the specimen. The specimen, in scattering the electrons, casts a large 'shadow' that can be viewed. However, we have so far only spoken of the electron as being attached to an atom; we must therefore describe, before going any further, how it is possible to obtain a beam of free electrons.

What do you think holds electrons to the atom?

It is the attractive force exerted by the positive charge on the nucleus that holds electrons to the atom. The different types of atom have electrons which can be detached with varying degrees of ease. For example, it is found to be comparatively easy to detach electrons from atoms of metallic substances such as iron or silver. This being the case, when these atoms are pushed close together, as they are in a solid, the outermost electrons in the atoms come within the sphere of influence of the electrical fields of neighbouring atoms. Some electrons are then attracted away from their parent nucleus to a neighbouring one. This may seem to contradict what we said earlier, namely that a charged body brought close to an atom would not experience any net force due to the atom. Previously we presented the argument that the forces exerted by the electrons on the one hand and the nucleus on the other would cancel each other out. It all depends on what we mean by 'close'. If the approach of the outermost electron is not too close (position A in Fig. 27), what we said earlier is true—its distance from the electrons of the neighbouring atom is about

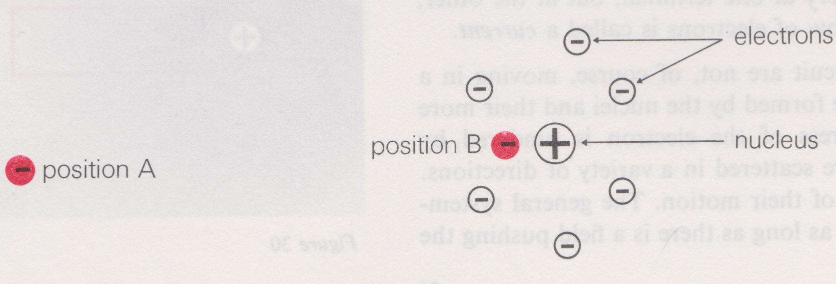


Figure 27

the same as its distance from the nucleus, so the forces are equal in magnitude but act in opposite directions. If the approach is exceedingly close however (say position B in Fig. 27), the difference in these distances becomes as large as the distances themselves and so can no longer be ignored—the forces exerted by the electrons and by the nucleus no longer cancel and a net force results. This is what happens in a solid. A loosely bound electron from one atom is able to get right in among the electrons of a second atom and thereby be attracted away from its original nucleus towards the neighbouring one.

It can then be attracted to the next one and so on. So a metal essentially consists of a lattice of nuclei surrounded by some electrons that are bound closely and tightly to their parent nuclei, and others that are free to wander around throughout the specimen. These nomadic electrons are called *conduction electrons* and are responsible for the way metals are good conductors of heat and of electricity.

Conduction electrons normally stay within the confines of the specimen of metal. This is because, when any tend to drift away, the specimen as a whole is left with an unbalanced positive charge that promptly attracts them back again.

The motion of the conduction electrons is usually quite random—there are as many moving in one direction as another, and throughout the specimen they are evenly distributed. If, however, the metal is placed in an electric field, the effect of the field will be felt throughout the interior of the metal (Fig. 28). The electrons within the metal will experience a force and those that are free to move in response to this force will do so. The result is that, in addition to the random motion, there will also be a general systematic drift in one direction due to the field (Fig. 29). This systematic movement of electrons cannot continue indefinitely because they come up against the surface of the specimen and this acts as a barrier to further movement (Fig. 30). If, however, the metal is in the form of a closed loop, say a length of wire, the movement of electrons can be maintained for as long as there is an electric field acting around the loop.

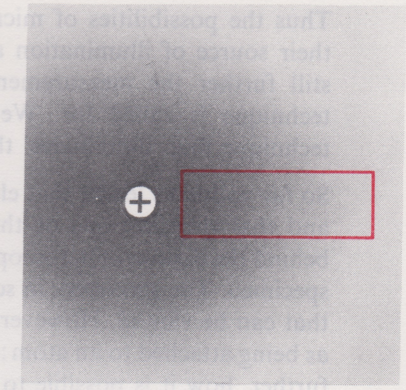


Figure 28

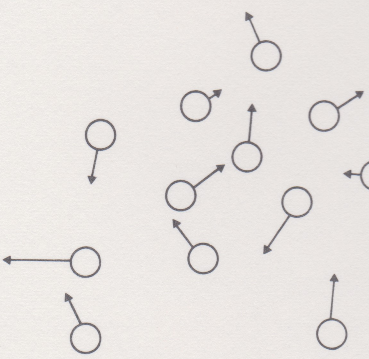


Figure 29 (*a*)

No electric field: random motion.

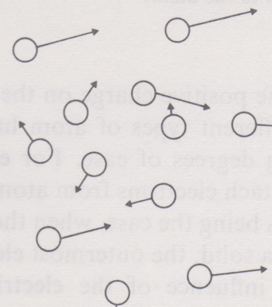


Figure 29 (*b*)

Electric field: random motion plus systematic drift.

A *battery* is a device for maintaining such an electric field. By connecting the ends of the wire to the terminals of the battery a *circuit* is established (Fig. 31). Electrons derived from the chemical reaction in the battery can flow through the wire, into the battery at one terminal, out at the other, and round the wire again. Such a flow of electrons is called a *current*.

The electrons passing round the circuit are not, of course, moving in a vacuum; they move through a lattice formed by the nuclei and their more tightly bound electrons. The progress of the electron is hindered by impurities in this lattice, and they are scattered in a variety of directions. This adds to the random component of their motion. The general systematic drift can only be maintained for as long as there is a field pushing the electrons along.

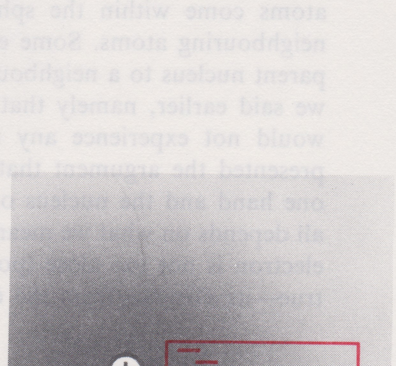


Figure 30

So the application of a field not only produces a current but also, because of this scattering, an increase in the random motion of the conduction electrons. And the faster on the average this random motion, the hotter the specimen is said to be. This is precisely what happens when one switches on an electric fire. A current is made to flow through a wire (called an element), this heats up and passes the heat out to the room.

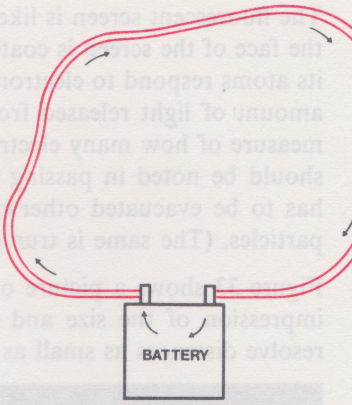
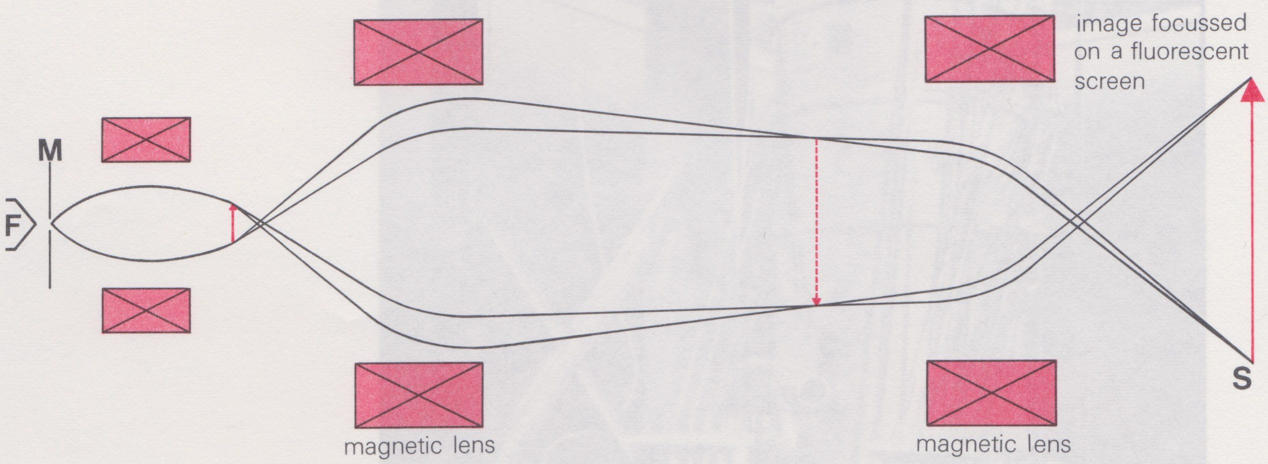


Figure 31

When a metal becomes very hot indeed (either by being heated electrically as above or by some other means) an interesting thing happens. The conduction electrons bounce around inside so fast that some of them are ejected completely from the specimen. To be sure, the unbalanced charge on the metal tries to pull them back, but if the speed of ejection from the surface is sufficiently great the electrons can get entirely away. It is rather like a projectile being launched from the surface of the Earth; normally gravity pulls it back, but if the speed of a rocket is sufficiently great it can escape the pull of gravity.

This effect provides a simple means for obtaining free electrons. Such electrons can be enormously useful. One of their applications is to be found in the *electron microscope*. Figure 32 (a) shows a schematic layout of such a microscope. F is a wire filament that is heated until it glows; it continuously gives off a cloud of electrons. These are immediately subjected to a powerful attractive electric field exerted by the charged metal plate M. The plate has a hole in it through which the accelerated electrons can pass. They are then directed on to the object that is to be viewed. The electrons scattered from the object are then focused on to a fluorescent screen S. The focusing is produced by magnetic fields which serve to alter the paths of the electrons in much the same way as optical lenses refract light in an ordinary optical microscope.* Comparison between Figures 32 (a) and 32 (b) shows how similar are the optics of the two types of microscope.



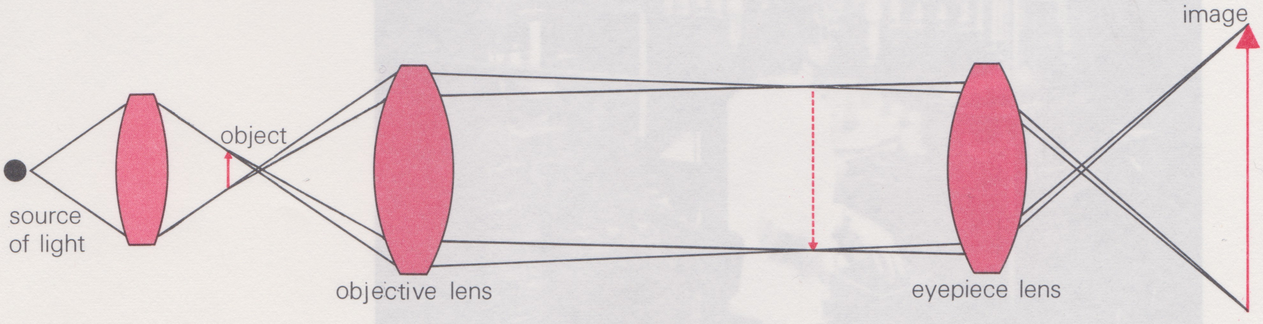


Figure 32 (*b*)

A comparison between the optics of (*a*) *an electron microscope and* (*b*) *an optical microscope.*

* *In due course you will have a more detailed explanation of the action of magnetic fields on moving charged particles* (*Unit* 32).

The fluorescent screen is like the one in your television set. The inside of the face of the screen is coated with a substance having the property that its atoms respond to electron bombardment by emitting visible light. The amount of light released from the various parts of the screen is then a measure of how many electrons are striking that region of the screen. It should be noted in passing that the interior of the electron microscope has to be evacuated otherwise the electrons would scatter from the air particles. (The same is true of a television tube.)

Figure 33 shows a picture of a modern electron microscope; it gives an impression of the size and complexity involved. Such instruments can resolve distances as small as about 2 or 3×10^{-10} m.

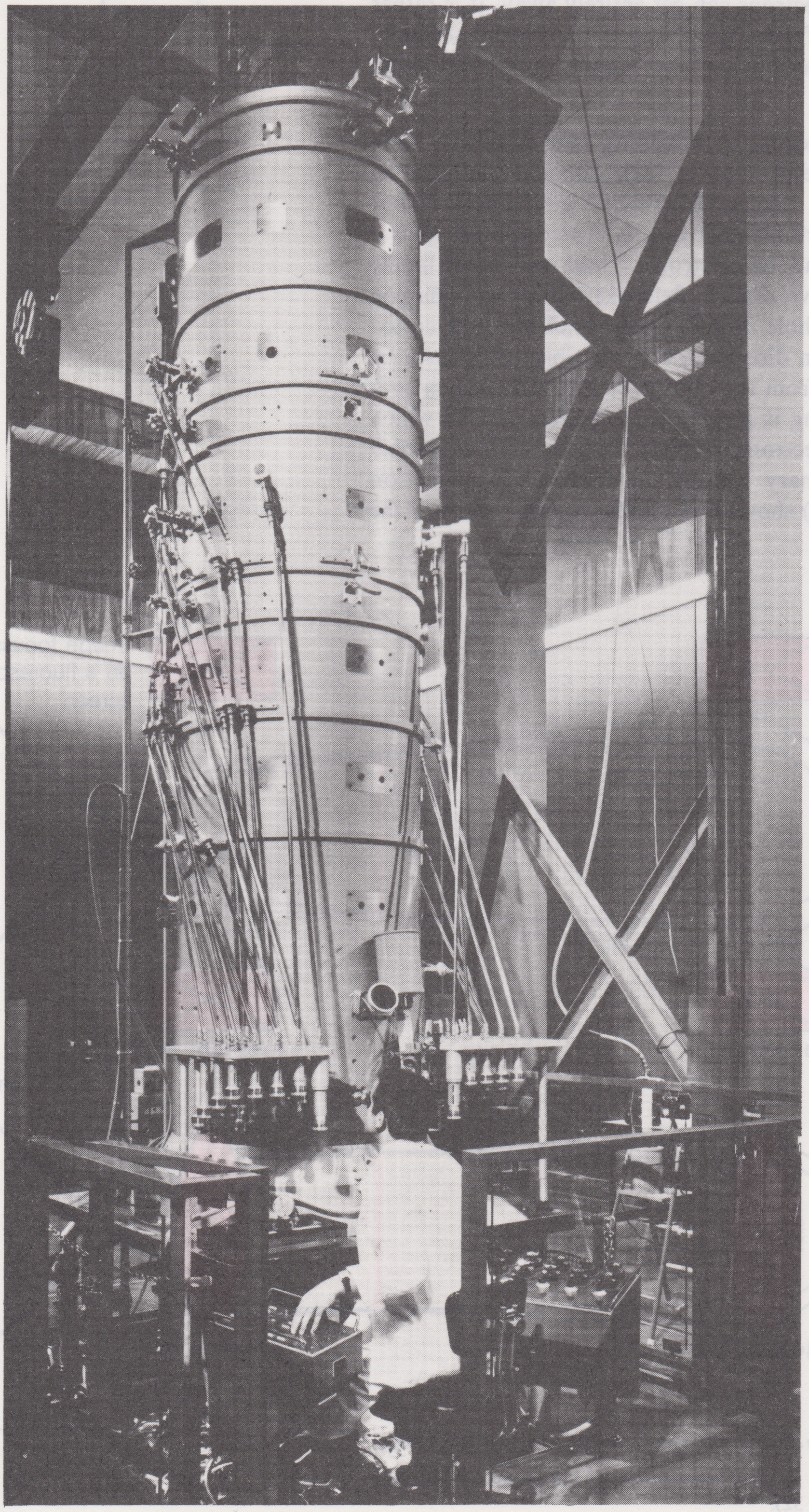


Figure 33

A large electron microscope at the Laboratoire d'Optique Electronique, Toulouse, France.

This is a very remarkable feat, but why, you may ask, stop there? If one is illuminating the specimen with a beam of particles, the type of limitation inherent in the use of waves has surely been overcome.

Now look at Figure 34. It is a photograph taken with an electron beam. What do you notice about it?

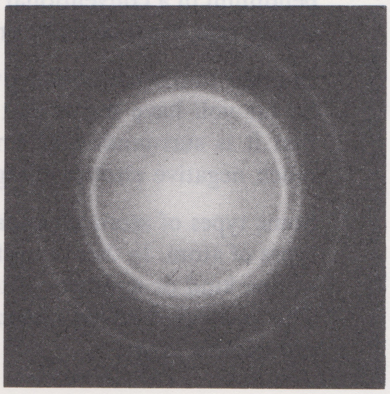


Figure 34

A photograph taken with a beam of electrons. You will see how this was done in the television programme of Unit 29.

In Figure 34 you come across a type of pattern you have seen before. With the ordinary optical microscope you saw that under high magnification the images became blurred and surrounded by circular bands.

What was the name given to the phenomenon that caused this?

So you see that electrons, which we have so far called 'particles', exhibit diffraction just like waves. This sets a limit to the ultimate resolving power that can be achieved with an electron microscope.* This ultimate limit is governed by the wavelength and this in turn is found to depend upon the momentum** of the electrons; the higher the momentum—the smaller the wavelength. The relationship is in fact a very simple one:

$$\lambda = h/p$$

Where λ is the wavelength of the electron, p is its momentum and h is a constant factor. So theoretically at least there is no limit to the resolving power achievable—ever smaller distances may be measured by using ever more energetic electrons. In practice, however, as in most modern sciences, there is the problem of finance—machines capable of accelerating electrons or other particles to great energies are expensive.

Later in Unit 29 we shall return to this topic of electron waves.

2.5.4 Measurement of long time intervals

One of the questions we shall be discussing in this Foundation Course is—how did the Earth evolve into its present state?

The Earth is constantly changing: rocks are eroded by rain and wind; sediments are deposited in the oceans, lakes and rivers; coastlines alter; earthquakes rend the surface; volcanoes build mountains; and it seems that even the floors of the oceans slowly spread and continents change their position.

These changes occur so slowly that virtually all of the Earth's major features were formed long before man existed. It is one of the tasks of the geologist to learn how the Earth has developed since its creation.

Geologists can usually tell whether one rock is younger than another by studying their spatial relationship. The simplest example of this is when two layers of rock, which were originally two layers of sediment, lie one on top of the other. It is obvious that the upper layer of sediment must have been deposited after the one below and is therefore younger; this is always providing that the beds have not been turned upside down by some violent contortion of the Earth. Similarly, by studying the fossils found in sedimentary layers, rocks of similar age in various parts of the world can be correlated and sequences of rocks, known as stratigraphic tables, drawn up.

* *We say 'ultimate' resolving power because in practice the actual resolving power of the instrument is more likely to depend upon its focusing properties.*

** *The momentum of a particle is defined as the product of its mass and velocity.*

However, neither of these types of investigation give any *absolute* idea concerning the age of rocks. To do this the geologist must find a naturally occurring process that takes place at a well-defined rate. This could then be used as the basis of a 'clock'—much in the same way as the swing of a pendulum in a conventional clock allows one to count off the seconds at a steady rate. Radioactivity provides such a process.

In order to discuss radioactivity, we must return to the study of the atom. So far you have learnt that the atom consists of a massive nucleus and that it is surrounded by tiny electrons. The electrons together carry as much negative electric charge as the nucleus carries positive charge.

Some types of atom are capable of spontaneously transforming into other types of atom. In these transformations the central nucleus emits radiations of one kind or another—often a positively or negatively charged particle. This process is called *radioactivity*, and one speaks of nuclei undergoing radioactive *decay*.

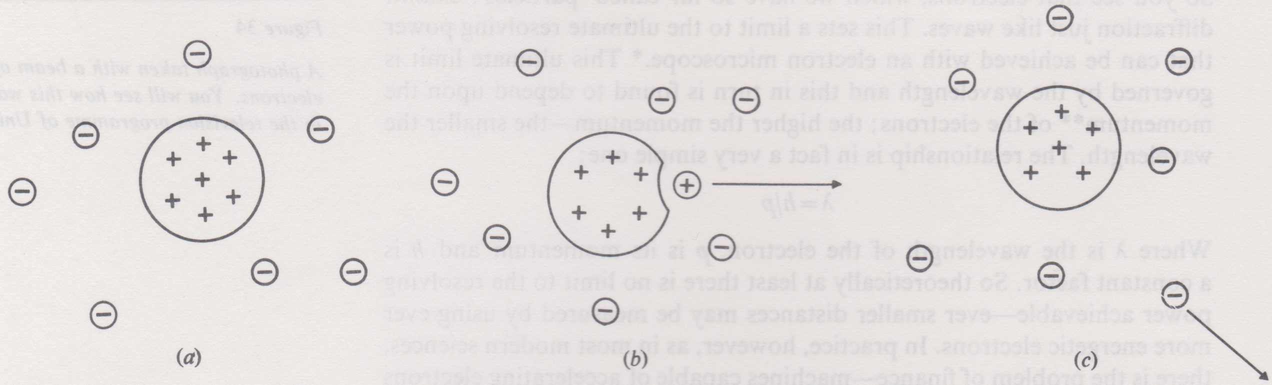


Figure 35

Radioactive decay: (a) An atom with seven negatively charged electrons and a nucleus carrying seven units of positive charge; (b) the nucleus undergoes radioactive decay by emitting a positively charged particle, leaving six units of positive charge on the nucleus compared to seven units of negative charge on the electrons; (c) one of the electrons becomes detached, so leaving an atom with an overall charge of zero once again. Note that the atom in (c) differs from that in (a) both in the charge on the nucleus and in the number of electrons —it is an atom of a different element.

In all transformations in nature it is found that the total amount of electric charge remains constant—or more strictly speaking, the amount of positive charge less the amount of negative charge remains fixed. This is called the *law of conservation of electric charge*. The emission of a charged particle from the nucleus, as depicted in Figure 35, therefore, alters the charge on the nucleus and consequently gives the atom an overall electrical charge. This in turn leads to electrons being gained from or lost to the surroundings in order to rebalance the charge on the atom as a whole. The final result is that an atom of one element has transformed into an atom of another element.

You need not worry at this stage about the details—what particles are emitted from the nucleus, what elements are involved in the transformations, etc. We shall deal with these aspects in later course units. We shall concern ourselves here with the general principle underlying *radiometric dating*, and in so doing introduce you to the broad class of phenomena described by what are called *exponential functions*.

Suppose we have nuclei of a certain type, X, and these are unstable and transform to nuclei of another type, Y. These transformations occur at a certain rate. The time taken for half the X nuclei to decay to Y nuclei is called the *half-life* of the transformation. We can denote it by the symbol, $T_{\frac{1}{2}}$, and use it as a convenient measure of the speed with which the transformation proceeds.

The half-life not only specifies the time taken to reduce an initial sample of X-nuclei to a half, but applies equally to the behaviour of any subsequent sample. Supposing the number of X nuclei present initially to be N, one would expect $N/2$ to remain after a time $T_{\frac{1}{2}}$. This number will itself be reduced by a half in a further period $T_{\frac{1}{2}}$. Thus after $2T_{\frac{1}{2}}$ one expects $N/4$ X nuclei to be left; after $3T_{\frac{1}{2}}$, $N/8$ nuclei, etc.

The numbers remaining in general will not be *exactly* equal to $N/2$, $N/4$, $N/8$, etc. This is because they will be subject to statistical fluctuations. For any particular nucleus there is no way of predicting when exactly the transformation will take place. One is only able to speak in terms of probability; thus the probability, or chance, of a particular nucleus decaying in a time period equal to a half-life is 50 per cent. As it is impossible to predict for a given nucleus when it will decay, it is clearly also impossible to know how many nuclei in a given sample will have decayed in a half-life. All one can say is that it should be about a half and the bigger the sample the closer it is likely to be to a half.

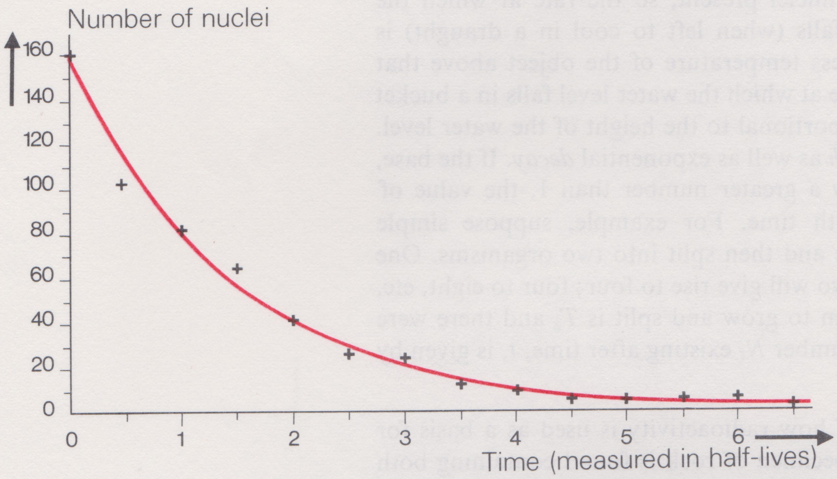


Figure 36

An exponential decay curve.

In Figure 36 we show the results of an experiment in which there were initially 160 nuclei. The smooth theoretical curve has been drawn to show that after one half-life we expect 80 nuclei to be left, after two half-lives we expect 40, etc. This theoretical curve is called an exponential decay curve. Following the same general trend, but not necessarily lying exactly on the curve, are the experimental readings.

You should now do the home experiment (Unit 2, Experiment 2).

Would you expect $T_{\frac{1}{2}}$ to depend upon the past history of a given sample of X nuclei? For example, would you expect it to depend upon how long the nuclei have already existed?

No. Since, at any given stage, it takes the same period $T_{\frac{1}{2}}$ to reduce the size of the sample by a further half, the rate at which the reaction occurs in no way depends on the past history of the particles. $T_{\frac{1}{2}}$ is also unaffected by extremes of temperature or pressure, to which radioactive materials occurring naturally in the Earth may have been subject in the past. Such extreme conditions affect only the electron configuration in an atom; much greater energies are involved in any alteration to the nucleus itself.

From the above we can deduce a formula relating the original number of X nuclei, N, to the number, N_t, remaining after any chosen time interval, t. This is:

$$N_t = N(\tfrac{1}{2})^{t/T_{\frac{1}{2}}} \qquad (3)$$

Check that this formula gives the same results as before by substituting successively the time intervals mentioned earlier, viz. $t=T_{\frac{1}{2}}$, $t=2T_{\frac{1}{2}}$, $t=3T_{\frac{1}{2}}$. Refer if necessary to *MAFS*, section 5.C.6.
If there are initially 1 600 X nuclei and the half-life is 3 hours, how many X nuclei would be expected to remain after 12 hours?

$N=1\,600$, $t=12$, and $T_{\frac{1}{2}}=3$, so from equation (3):

$$N_t=1\,600\,(\tfrac{1}{2})^{12/3}$$
$$=1\,600\,(\tfrac{1}{2})^4$$
$$=1\,600\,(\tfrac{1}{16})=100$$

$\therefore$ the number remaining is 100.

The expression on the right-hand side of equation 3 is called an *exponential function.*

Can you guess why?

This is because the quantity that is changing (i.e. the variable) which in this case is the time, t, appears as an exponent.*

Though we have so far spoken only of radioactivity, you should note that there are many types of physical behaviour described by expressions of this form. Just as the number of nuclei decaying in a given time is directly proportional to the number of nuclei present, so the rate at which the temperature of a heated body falls (when left to cool in a draught) is directly proportional to the excess temperature of the object above that of its surroundings. Also, the rate at which the water level falls in a bucket with a hole in the bottom is proportional to the height of the water level. One may have exponential *growth* as well as exponential *decay*. If the base, '$\frac{1}{2}$', in equation 3 is replaced by a greater number than 1, the value of the expression will increase with time. For example, suppose simple organisms grow to a certain size and then split into two organisms. One organism will give rise to two; two will give rise to four; four to eight, etc. If the time taken for an organism to grow and split is T_2 and there were initially N organisms, then the number N_t existing after time, t, is given by $N_t=N2^{t/T_2}$.

We now turn to the question of how radioactivity is used as a basis for time measurement. Suppose a specimen of rock is found containing both X and Y atoms, and that there are reasons for assuming that all the Y atoms have been created as a result of the disintegration of X atoms (i.e. no Y atoms were present when the rock was formed), and also that the ratio of X atoms to Y atoms has not been affected by any preferential loss of either X or Y to the surroundings. (This is known as a closed system.) Knowing the characteristic half-life for the reaction X→Y (from a separate experiment carried out on another specimen of X), and having determined by analysis the numbers of the atoms of both types present in the rock, one can calculate what length of time was required to produce the ratio of X to Y.

As a simple illustration, suppose the reaction to be characterized by a half-life of 5 500 years and that for every X atom there are now 15 Y atoms. This means that originally there were 16 X atoms where now there is only one. The other 15 X atoms have decayed to Y atoms in a certain time we shall call t. Then in equation 3, $N_t=N(\frac{1}{2})^{t/T_{\frac{1}{2}}}$ ....(3)

$$N_t=1,\ N=16,\ T_{\frac{1}{2}}=5\,500 \text{ years}$$
$$\therefore\quad 1=16\,(\tfrac{1}{2})^{t/5500}$$
$$\text{hence}\quad \tfrac{1}{16}=(\tfrac{1}{2})^{t/5500}\quad \text{But } \tfrac{1}{16}=(\tfrac{1}{2})^4$$
$$\therefore\ (\tfrac{1}{2})^4=(\tfrac{1}{2})^{t/5500}$$
$$\text{hence}\quad 4=t/5500\quad \therefore\ t=4\times 5\,500 \text{ years}$$
$$t=22\,000 \text{ years}$$

* *An exponent is the number indicating the* power *of a quantity. For example the exponent of* x *in* x^4 *is* 4. *(See* MAFS, *section* 1.A.1.)

In a more complicated example (one in which t is not arranged to be a convenient integral multiple† of $T_{\frac{1}{2}}$) you would have to solve equation (3) in a more general way:

$$N_t = N(\tfrac{1}{2})^{t/T_{\frac{1}{2}}}$$

$$\therefore \quad N_t/N = (\tfrac{1}{2})^{t/T_{\frac{1}{2}}}$$

$$\therefore \text{ Inverting } \quad \frac{N}{N_t} = 2^{t/T_{\frac{1}{2}}}$$

Taking logarithms of both sides (if necessary refer to *MAFS*, section 1.B).

$$\log (N/N_t) = \frac{t}{T_{\frac{1}{2}}} \log 2$$

$$\therefore \quad t = \frac{T_{\frac{1}{2}} \log (N/N_t)}{\log 2} \quad \text{......(4)}$$

If you use logarithms to the base 10

$$\log_{10} 2 = 0.3010,$$

$$\frac{1}{\log_{10} 2} = 3.322$$

$$t = 3.322\, T_{\frac{1}{2}} \log_{10} (N/N_t) \quad \text{......(5)}$$

A nucleus rubidium-87 (^{87}Rb) decays to another, strontium-87 (^{87}Sr) with a half-life of 4.7×10^{10} years. In a certain mineral 0.680 per cent of its weight was found to be due to its content of ^{87}Rb and 0.015 per cent to ^{87}Sr. Assuming the masses of the two nuclei to be approximately equal and that no ^{87}Sr was present when the rock was formed, calculate the age of the rock.

As the nuclei can be assumed to be of equal mass the ratio of the percentage weights is the ratio of the number of nuclei present.

$$N/N_t = \frac{0.680 + 0.015}{0.680} = 1.022$$

$$\log_{10}(N/N_t) = \log_{10} 1.022 = 0.0095$$

From equation (5), using $T_{\frac{1}{2}} = 4.7 \times 10^{10}$ years

$$t = 3.322 \times 4.7 \times 10^{10} \times 0.0095$$
$$\approx 1.5 \times 10^9 \text{ years}$$

Studies such as these can be used to estimate the age of minerals and rocks. For example, it has been calculated that the volcanic rocks forming much of North-West Scotland are between 50 and 60 million years old, whereas the granites of Cornwall, which have been worked for years for their tin and china clay, are about 400 million years old.

The oldest rocks found in the Earth's crust are 3.4×10^9 years old. The Earth must therefore be at least as old. In fact, the Earth is thought to be considerably older than this; it probably originated 4.6×10^9 years ago. If the oldest rocks are 3.4×10^9 years there is obviously a gap of 1.2×10^9 years. We shall discuss this difference in Units 26 and 27.

As we said earlier, the fossil remains of animals and plants are to be found in various rocks. Having measured the age of rocks by radiometric means, one can determine the age of the fossils. Some of the very oldest rocks contain the organic chemical substances,† amino-acids—the building bricks of life (you will learn more of these later in Units 10 and 14). But fairly complicated animals first appeared 0.6×10^9 years ago (some of these are still to be found on present sea-shores virtually identical to their ancient ancestors). Compare this period to man's span on Earth—a mere one million years during which time he has evolved from a recognizable though rather hairy ancestor!

If you wish you may now turn to Appendix 6 (Black) on exponential functions. This pursues the topic of an exponential function a little further; it introduces you to the idea of a *mean-life* and also to the base, 'e'.

2.5.5 Measurement of short time intervals

Finally we turn to the measurement of short time intervals. You saw in our discussion of photography that one way to see the details of fast-changing processes was to use high-speed cinematography. In this technique, the camera shutter is kept open and the film moves continuously. The experiment or process to be recorded is periodically illuminated by intense flashes of light of very short duration. These flashes are repeated at regular intervals and the successive images are formed on the film alongside each other. Figures 37 and 38 show two examples of rapidly changing phenomena photographed by short-duration flashes. The explosion of Figure 39 provided its own intense illumination, so in this case the technique was to use a fast shutter to expose the film for only a short time.

Counting the number of pictures taken between the beginning and the end of a sequence, and knowing the rate at which the pictures are taken, one can calculate the time interval that has elapsed. Modern high-speed cameras can take pictures at the rate of several million per second; by this method therefore it is possible to resolve time intervals of a fraction of a millionth of a second.

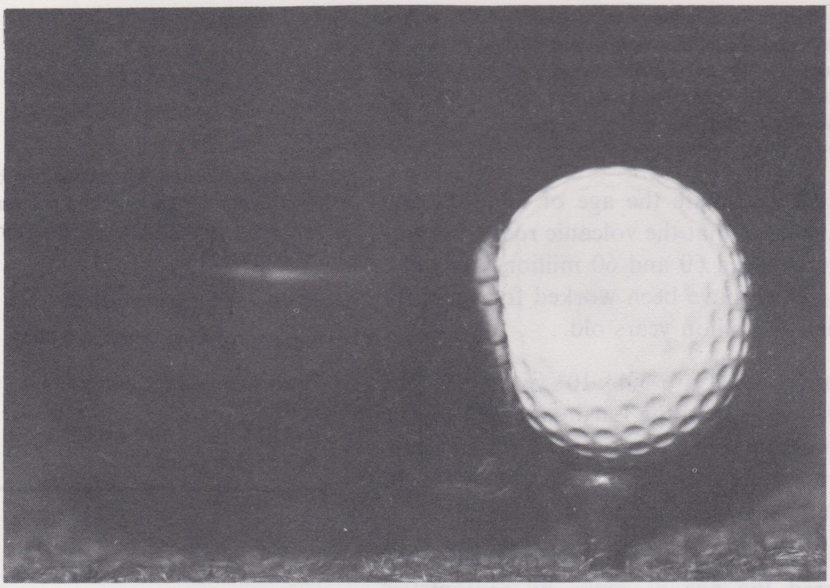


Figure 37

A golf ball deformed at the moment of impact.

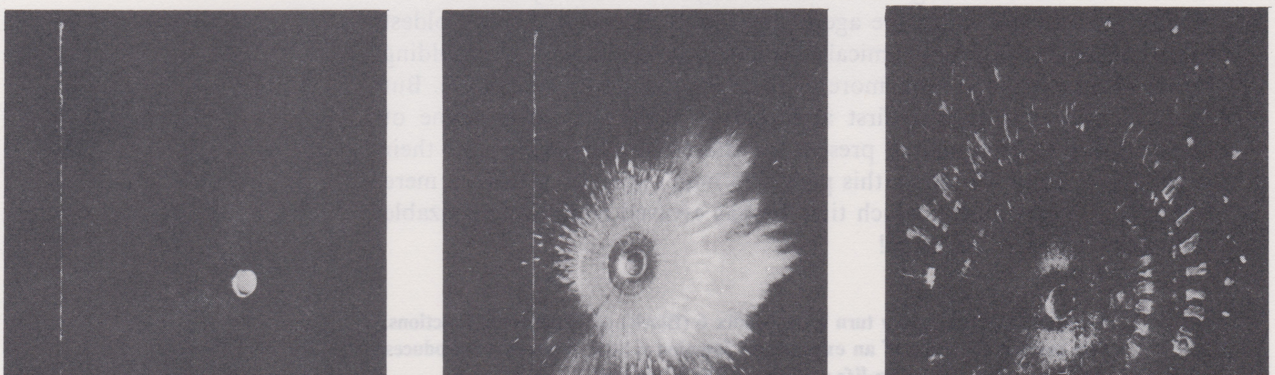


Figure 38

A bullet striking a steel block. The lead in the bullet apparently liquefies, splatters, and then resolidifies.

The *cathode-ray oscilloscope* provides another approach to the problem of measuring short time intervals. This instrument is very versatile and is widely used in many laboratories. In order to understand its detailed construction one has to have a knowledge of electronic circuits. Nevertheless it is not difficult to appreciate the basic principles of its operation.

In Figure 40 we show a simplified diagram of its construction. On the left is a wire filament through which a current is passed. This heats a metal plate called a cathode, which emits the electrons that are to be accelerated. The rate at which the electrons are accepted for acceleration could be modified by making changes to the temperature of the cathode, but in practice it is more convenient to have a metal grid with holes in it called a control grid. Depending on how one charges the grid relative to the cathode one is able to vary the electric field in the immediate vicinity of the cathode. This in turn affects the number of electrons passing through the holes in the grid.

The electrons passing through the grid tend to be moving in various directions; the purpose of the next component therefore is to focus the beam. Acceleration is produced by the accelerating anode (the word 'anode' meaning that it is electrically positive with respect to some other component—the 'cathode').

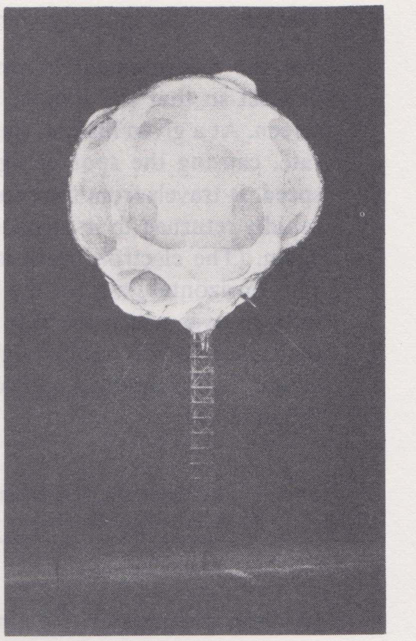


Figure 39

A nuclear bomb explosion at an early stage.

The electrons on emerging through a hole in this anode pass through two pairs of parallel plates XX′ and YY′, each pair being at right-angles to the other. If an electric field is established between X and X′, the beam will be deviated horizontally, the direction and magnitude of the deflection depending on the direction and magnitude of the field. Likewise, an electric field between plates Y and Y′ produces vertical deviation. Therefore, a suitable combination of electric fields across XX′ and YY′ directs the beam to any desired point on the screen.

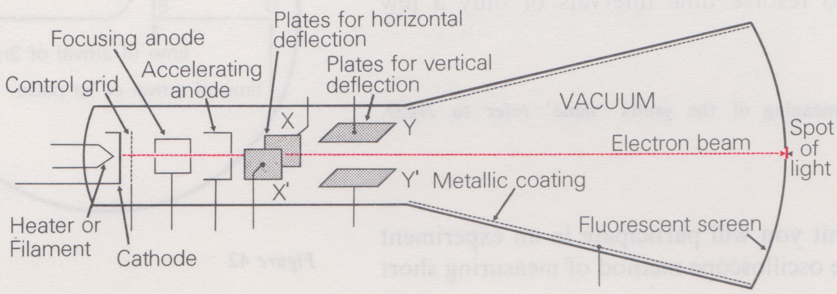


Figure 40

Simplified diagram of a cathode-ray tube.

On reaching the screen, the electrons cause the coating on the inside of the screen to emit visible light. The principle, as you can see, is very similar to that of the electron microscope. The brightness of the spot depends on the speed of the electrons and also on the number of electrons arriving at that point.

What controls this number?

This number can be adjusted by altering the field between the control grid and the cathode.

What component controls the size of the spot?

The size of the spot is governed by the focusing anode.

For the measurement of time intervals, the field between X and X′ is first set so that the spot of light is at the extreme left-hand edge of the screen. At a given instant, the field across XX′ begins to change at a steady rate, causing the spot of light to move across the screen at a constant speed. It travels from the extreme left to the extreme right. The field is then rapidly returned to its original value, taking the spot to the left-hand side again. (The electron beam is suppressed so that its 'return' is not seen.) This horizontal path for the spot of light is called the *time-base*. The time interval over which the field on XX′ is allowed to change is the same as that taken for the spot of light to perform one complete sweep across the screen. By inscribing a scale on the screen one may subdivide the length of the time-base.

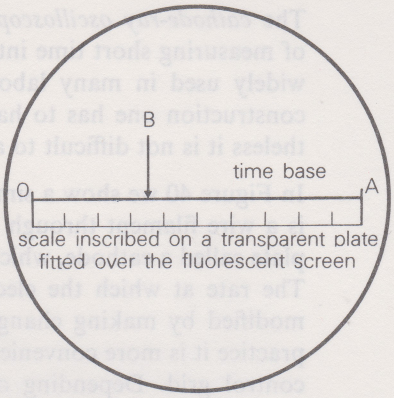


Figure 41

In Figure 41, if it takes the spot 10^{-6} seconds to traverse the complete time-base, OA, estimate how long it takes to reach the point B.

OB reads 3.5 divisions and OA 10 divisions. ∴ The time for the spot to reach B is $(3.5/10) \times 10^{-6} = 0.35 \times 10^{-6}$s

In order to measure the time taken to complete a given process, it is necessary to arrange for two electrical signals to be sent to the oscilloscope —one at the time the process is initiated and another when it is completed. The first pulse signals to the oscilloscope that it should start sweeping the spot across the face of the screen (we say it 'triggers the time base'). The other is applied to one of the plates of the YY′ pair so as to produce a sudden change in electric field in the vertical direction. Because electrons have such a small mass, their motion is easily altered by forces acting upon them. So the electron beam rapidly responds to the force exerted on it by the change in the vertical field, and the spot of light on the screen deviates from its hitherto steady horizontal motion. In Figure 42 you see a typical trace. Clearly by measurement of the distance OB one can calculate the time between the two signals.

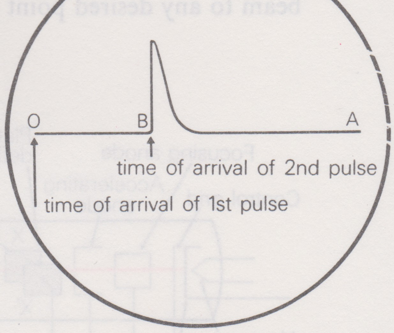


Figure 42

By this method it is possible to resolve time intervals of only a few nanoseconds.

If you have forgotten the meaning of the prefix 'nano' refer to *HED*, section 3.1.5.

In the TV component of this Unit you will participate in an experiment in which you will make use of the oscilloscope method of measuring short time intervals.

Your TV set is very similar in construction to an oscilloscope and to a limited extent can be used as one.

Try home-kit experiment (Unit 2, Experiment 3).

This concludes our brief survey of some of the ways in which measurements of length and time are being extended. In the next Unit you will be studying more closely what exactly one means by the terms 'length' and 'time'.

Appendix 1 (White)

A Short Description of the Principles underlying the Photographic Technique

Certain substances react to light—that is why fruits ripen and fabrics fade in sunshine. Silver bromide is the name of a chemical compound that responds rapidly to exposure to electromagnetic radiation. A compound is a substance that is made of two or more atoms combined in fixed proportions. Silver bromide consists of atoms of silver and of bromine combined in the ratio one-to-one.

When atoms of different types join together, the physical properties of the resulting compounds may differ markedly from those of the separated constituent atoms. Silver bromide is a light-coloured crystalline substance, whereas silver on its own is, of course, a shiny opaque metal, and bromine is a red-brown liquid.

The effect of electromagnetic radiation falling on a silver bromide crystal* is to cause a few electrons to leave their parent atoms. They are then believed to bounce around inside the crystal until they eventually become trapped, usually at the crystal's surface. There they congregate and, because of their electric charge, succeed in disrupting certain of the bonds binding the silver and bromine atoms. The silver atoms, on being released from their bromine atoms, revert to the normal behaviour characteristic of silver, i.e. they become specks of opaque metal.

In the photographic technique, a great many of these tiny crystals are embedded in gelatine and this so-called 'emulsion' is coated on to a strip of celluloid (or on to a glass plate) which acts as a supporting base. Wherever light falls on this photographic emulsion tiny specks of silver are deposited within the crystals. This distribution of specks is called the *latent image*.

Unfortunately this image is very faint—only a few atoms of silver may have been deposited in each crystal. The emulsion is, therefore, treated with a chemical called a *developer*. The action of the developer is to free further silver atoms. This is normally a very slow process taking perhaps several hours. Where, however, there are already present in the crystal a few specks of metallic silver, these act as centres from which the deposition of silver quickly spreads. In this way the process is enormously speeded up and it takes only a matter of a few minutes to develop the whole crystal, i.e. deposit its silver atoms as metal.

Figure 43 shows a highly magnified ($\times 2500$) picture of an emulsion that is only partly developed. You can see quite clearly the metallic silver being deposited around centres located on the surfaces of the crystals.

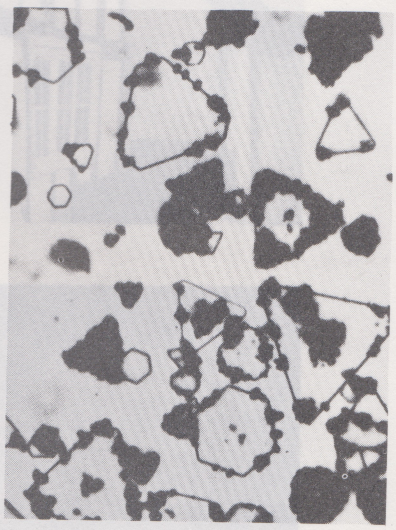


Figure 43

Silver bromide crystals at an early stage of development showing metallic silver being deposited at the crystal surfaces.

The process is rather analogous to the onset of an avalanche. Initially a single small stone might be dislodged and this of itself might seem insignificant. If, however, the conditions are right, it can quickly lead to whole rocks being dislodged. In the photographic process it has been calculated that, for each initial metallic silver atom present in the latent image, there are 10^9 after development.

The period for which the emulsion is allowed to remain in the developer

* *A crystal is a quantity of substance consisting of atoms arranged in a regular pattern in space. Though there exist liquid crystals, most are solid bodies of regular shape bounded by plane surfaces. Most solids are composed of crystals.*

is chosen so that most of the crystals that were exposed to the radiation are developed, and only a few of those that were unexposed.

After development the emulsion is immersed in another chemical called a *fixer*.

What would you guess was the purpose of this?

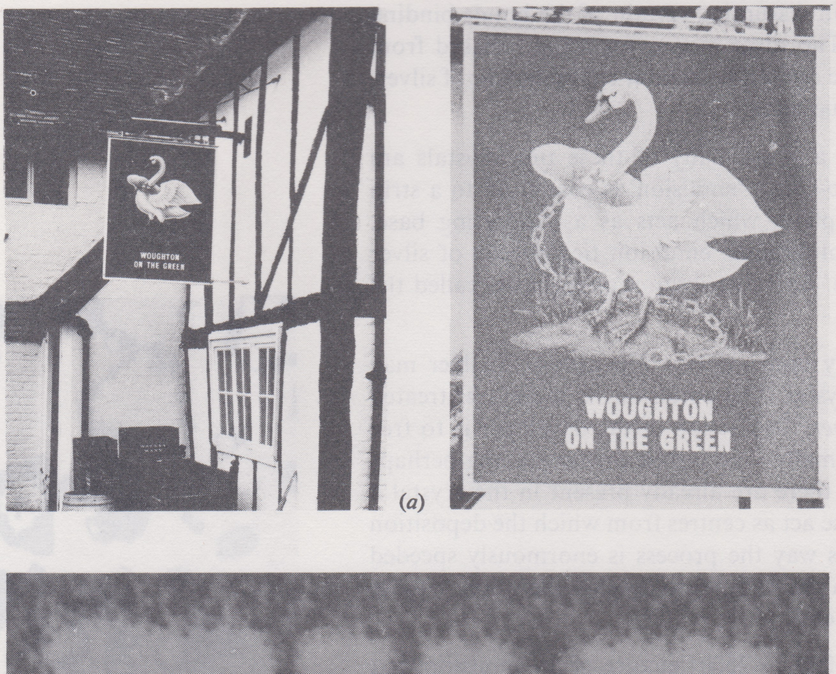
It removes the residual silver bromide. This is necessary, otherwise subsequent exposure to light would cause further darkening.

The process we have described produces a 'negative', i.e. a picture where all the light areas show up dark and *vice versa*. To produce a 'print' from the negative one has to reverse the light and dark areas once more.

This is done by shining light through the negative and onto a sheet of paper coated with emulsion. This in turn is developed and becomes the final picture.

If the photograph of Figure 44 is looked at under progressively greater magnifications, the grainy structure of the picture becomes obvious (Figs. 44 (b) and (c)).

We have spoken only of silver bromide; there are indeed other chemicals that are sensitive to electromagnetic radiation. Various of these chemicals or combinations of them can be made to give emulsions sensitive to radiation over different ranges of wavelength.



(a) (b) (c)

Figures 44 (a), (b), and (c)—a photograph seen under successively greater magnification; showing its grainy structure.

Appendix 2 (White)

The Principle of the Microscope

The microscope is a device for enabling one to see magnified images. In its simplest form it consists of a single lens. This is called a simple microscope or more commonly a *magnifying glass*.

The closer an object is to the eye the bigger it looks. But if it is brought too close, the lens of the eye cannot refract the rays sufficiently to focus them on the retina.* A lens placed in front of the eye introduces additional bending of the rays; this enables focusing to be achieved even with the object quite close (Fig. 45).

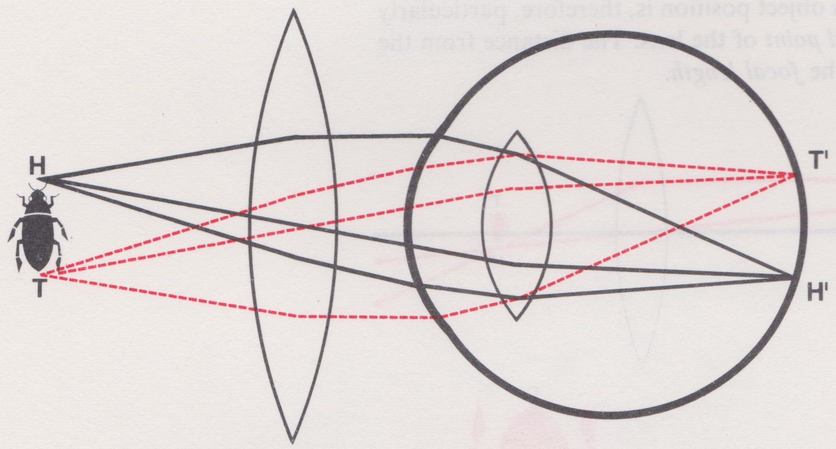


Figure 45

Diagram showing how a magnifying glass supplements the focusing properties of the eye itself.

You are supplied in your home experimental kit with a lens. Use it as a magnifying glass. Note that the image you see through it is the 'right-way-up'—in spite of the image on the retina being inverted (Figure 45). If you do not understand why this is so refer to Appendix 5.

Auxiliary lenses are, of course, also used in spectacles to correct defective vision.

The magnifying glass has its limitations. For one thing there is a limit as to how close the object can be brought up to the eye, due simply to the physical dimensions of the auxiliary lens.

At a further level of sophistication, one has the *compound microscope* (normally just referred to as 'the microscope'). The principle of the microscope is that a lens called the *objective lens* produces a magnified image of the object. The magnifying glass, instead of being used to look at the object directly, now views this already magnified image.

* *This situation is shown in Figure* 60, *Appendix* 5, *p.* 56.

Figures 46 (a), (b), (c), (d), show what happens to the rays passing through a lens when the object is brought up closer to it.

Roughly, how do you expect the light rays to be for Figure 46d?

For the answer turn to Figure 46 (e), at the foot of p. 51, where the rays are drawn.

You see from this series of figures that the closer one brings the object, the further away the image is formed and the bigger it becomes. In Figure 46 b, by the similar triangles HOL and H′O′L (see *MAFS*, section 2.E, if necessary):

$$\frac{\textit{Size of image}}{\textit{Size of object}} = \frac{\textit{distance of image to lens}}{\textit{distance of object to lens}} \qquad \text{......................................}(6)$$

A stage is reached however (Fig. 46 c) when the lens is no longer capable of making the rays converge. This object position is, therefore, particularly noteworthy and is called the *focal point* of the lens. The distance from the focal point to the lens is called the *focal length*.

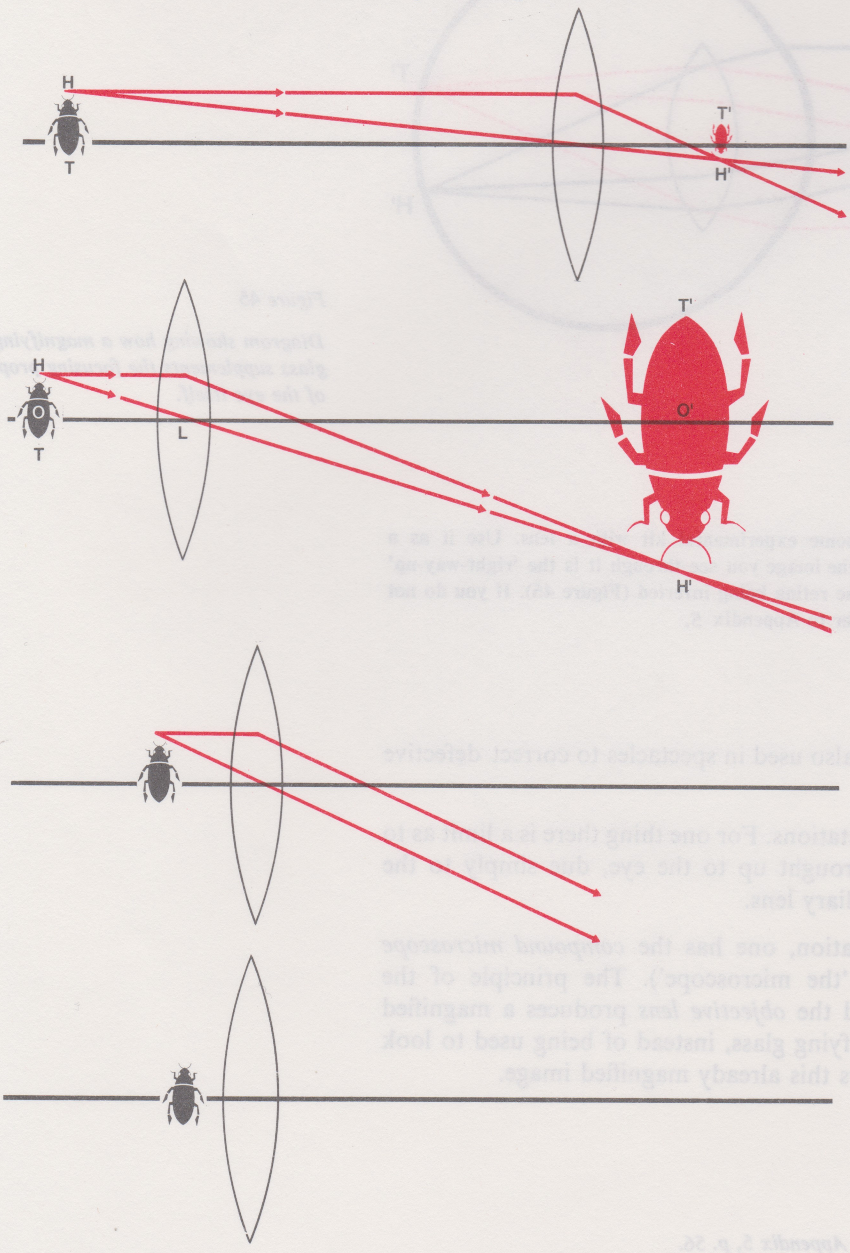


Figure 46

So you see that if the object is placed just a little further back from the lens than its focal point one obtains a large *inverted image*. To see this image one has only to place a screen at the position of the image (Fig. 47).

Alternatively one can look at it with a magnifying glass as in Figure 48. Here one has, in fact, combined the arrangements given in Figures 45 and 46 b. The lens nearest the object is the *objective lens* and the other (our original 'magnifying glass') the *eye-piece lens.*

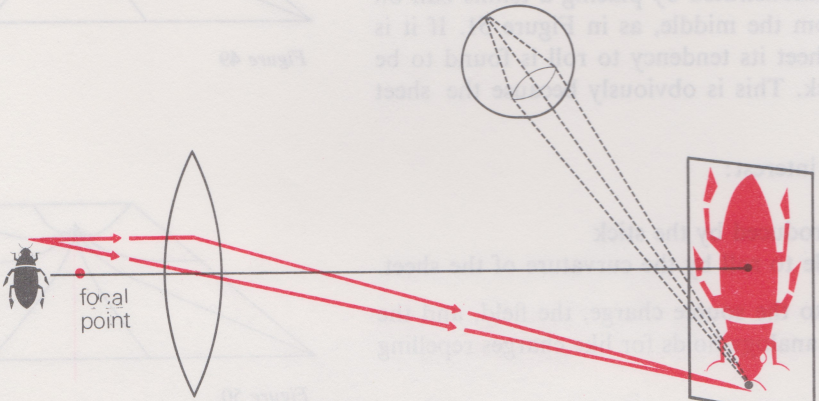


Figure 47

Diagram showing that an object placed a little further from the lens than the focal point produces a large inverted image.

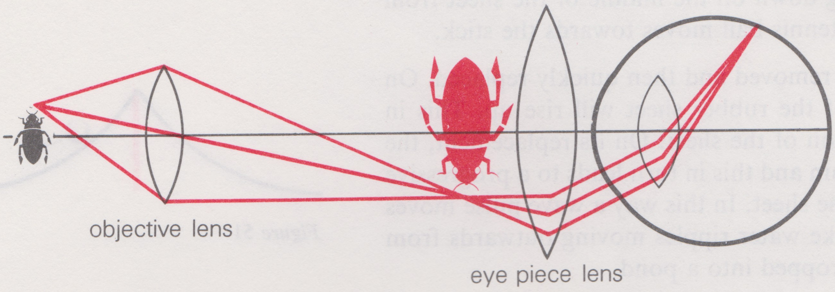


Figure 48

The optics of the microscope.

Figure 48 shows the paths of the rays of light as they pass through the microscope and into the eye.

From equation 6 you see that in order to increase the size of the final image one can either increase the image distance or decrease the object distance to the lens. The first alternative is not of much help. If the microscope is to be at all convenient to handle it cannot have an enormously extended length. The second alternative is the more promising—the object is brought up closer to the objective lens.

Are there any restrictions on how close it can be brought?

Recalling that the object must be kept further away from the lens than the focal point it is clear that in order to reach the very highest magnifications it is necessary to have very thick lenses with short focal lengths. This is why the more expensive microscopes have a selection of objective lenses permitting different magnifications.

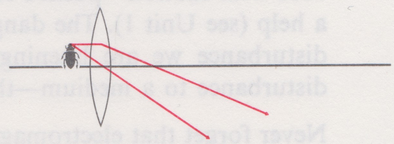


Figure 46 (e)

Answer to question posed at head of p. 50.

Appendix 3 (Red)

Fields

Let us see if the following analogy can help you get a better feel for this concept of a *field*.

Imagine a very large rubber sheet stretched out tight as in Figure 49. Initially it is flat. Then the middle of the sheet is pushed up with a stick as in Figure 50. The stick is seen to affect the whole sheet—the sheet is now in a different 'condition' to what it was when it was flat.

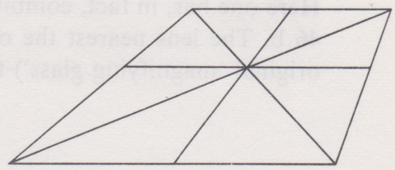


Figure 49

This changed condition can be demonstrated by placing a tennis ball on the sheet. The ball rolls away from the middle, as in Figure 51. If it is placed at various points on the sheet its tendency to roll is found to be greater the closer it is to the stick. This is obviously because the sheet is steeper close to it.

Note there are three quantities of interest:

(1) the stick
(2) the curvature of the sheet produced by the stick
(3) the tennis ball which is made to roll by the curvature of the sheet.

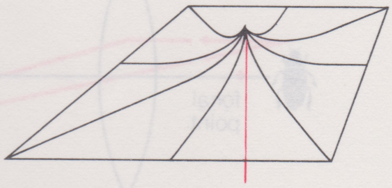


Figure 50

These are analogous respectively to the source charge, the field, and the body acted upon by the field. The analogy holds for like charges repelling each other.

For the case of unlike charges attracting each other, the stick can be replaced by a second one pressing down on the middle of the sheet from above, as in Figure 52. Now the tennis ball moves towards the stick.

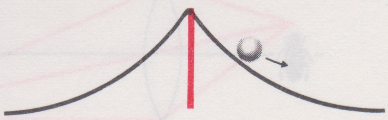


Figure 51

Suppose the stick is momentarily removed and then quickly replaced. On its removal, the centre portion of the rubber sheet will rise up. This in turn affects an ever-widening region of the sheet. On its replacement, the centre portion is pushed down again and this in turn leads to a progressive lowering of the other regions of the sheet. In this way a wave pulse moves outwards over the sheet, rather like water ripples moving outwards from a point where a stone has been dropped into a pond.

Note that after the stick has been replaced one cannot, from a study of the stick alone, see all that is going on—one cannot predict that at some subsequent time a distant tennis ball lying on the sheet is going to receive a jolt. The behaviour of this ball can only be explained by reference to the travelling distortion on the rubber sheet.

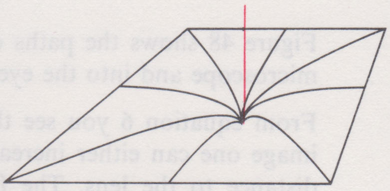


Figure 52

This is analogous to our source charge being moved from one position to another and back again; to understand the subsequent movement of a distant charge one must concentrate attention on the field.

Warning

Analogies are all very well and this one may serve to give you a helpful mental picture for visualizing these fields. However, analogies must be used with caution—pushed too far they become more of a hindrance than a help (see Unit 1). The danger with this particular analogy is clear. The disturbance we are likening our electromagnetic wave to is in fact a disturbance to a medium—the rubber sheet. No rubber sheet—no wave.

Never forget that electromagnetic waves do not require a medium.

Appendix 4 (Red)

Lenses

In Figure 53, light is diverging in all directions from a source S. Some will pass from the source to a given point A on the surface of a glass block. SA is one of the many possible paths that light emitted from S can take—it is called a *ray*.

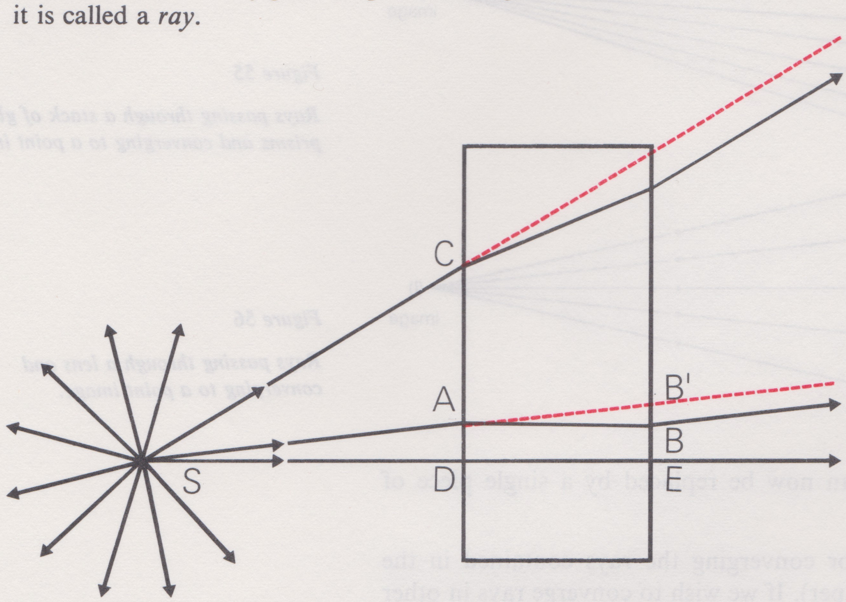


Figure 53

Diagram showing rays of light from source, S, being refracted on passing through a parallel-sided glass block.

On striking the surface of the glass block the ray of light changes direction, bending towards the normal† to the surface, and emerges at point B and not at B′. If the block is parallel-sided the ray will suffer a similar change in direction at this second face but in the opposite sense, away from the normal. The net effect is that the ray emerges parallel to the initial ray but displaced laterally. Similar behaviour is found when the glass block is replaced by any other medium in which the velocity of the light wave changes on crossing the boundary. This change of direction in the light when passing from one medium into another is called *refraction*.

The amount of bending depends upon the angle at which the ray strikes the surface. This is illustrated by rays SC and SD. In the latter case, where it meets the surface at right-angles, i.e. normally, it passes on undeviated and emerges at E. If the block were not parallel-sided, the emergent rays in general would not only be laterally displaced but also show a resultant change in direction as in Figure 54.

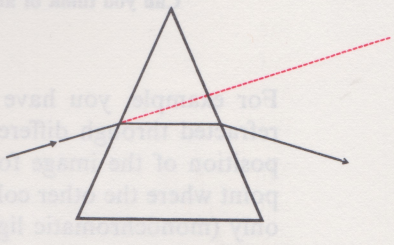


Figure 54

Diagram showing a ray of light emerging from a wedge-shaped glass block and having changed its direction.

What factors do you think govern the magnitude of this change of direction?

This overall change in direction will depend on the angle at which the ray strikes the first face, the angle between the faces, on the exact nature of the glass and on the wavelength of the light. Now imagine a set of glass prisms (i.e. wedge-shaped blocks of glass) stacked on top of each other, each with a slightly different angle between its opposite faces compared to that of its neighbour. These angles are arranged so that all the light rays coming from the source and passing through the stack reconverge to a single point, I, called the *image* (Fig. 55).

By making the individual prisms smaller and correspondingly increasing their number, the surfaces of the stack become smooth, as shown in Figure 56.

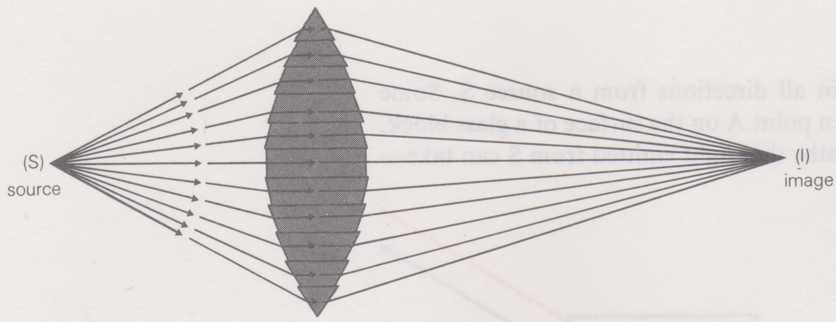


Figure 55

Rays passing through a stack of glass prisms and converging to a point image.

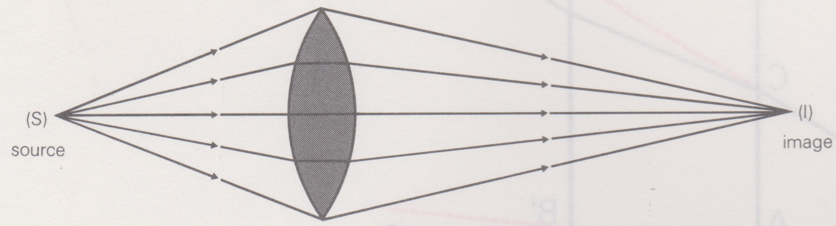


Figure 56

Rays passing through a lens and converging to a point image.

The stack of separate prisms can now be replaced by a single piece of glass called a *lens*.

This arrangement is all right for converging the rays contained in the plane shown (the plane of the paper). If we wish to converge rays in other planes as well then the lens must have the same profile in all planes—in fact it is found that both surfaces have to approximate to portions of *spheres*.

This is a very simple picture of the way in which a spherical lens forms images. In practice there are departures from this idealized situation; the rays do not all converge to a single point and the resulting defects are called *aberrations*.

Can you think of any reason why such defects are to be expected?

For example, you have just read how light of differing wavelength is refracted through different angles. If point I in Figure 56 is the correct position of the image for one component of the light it will not be the point where the other colours will converge. Even with light of one colour only (monochromatic light) there are a variety of aberrations depending on the position of S relative to the lens, and also on the size of the lens.

As lenses are an integral part of many optical instruments, for example the microscope and telescope, the reduction of aberrations to a minimum is one of the chief problems in their design.

Appendix 5 (Red)

The Optics of the Human Eye

The human eye is essentially an optical instrument. Light enters through the transparent *cornea* (Fig. 57) and in so doing is refracted. Its direction is further modified by the *lens*. The light is focused on to the sensitive inner layer of the rear of the eye called the *retina*. Muscles attached to the extremities of the lens serve to alter its shape and so vary its focusing properties. In front of the lens is a circular screen with an aperture of variable size at the centre. The screen is called the *iris* and is the part of the eye that shows as a coloured ring. The central aperture is the *pupil*. By varying the size of the pupil, the iris controls the amount of light admitted. Both between the cornea and the lens, and between the lens and the retina are watery fluids.

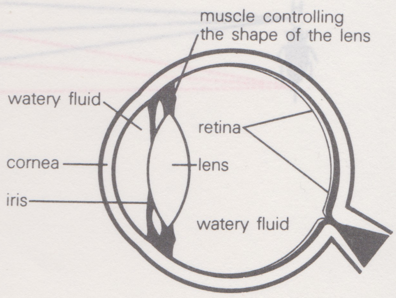


Figure 57

The human eye.

When an object is viewed, light diverging from a point on its surface will, through the combined action of the curved cornea and the lens, converge to a corresponding point on the retina.

Given the arrangement shown in Figure 58, roughly whereabouts on the retina would you expect to find the images of the head and tail of the insect?

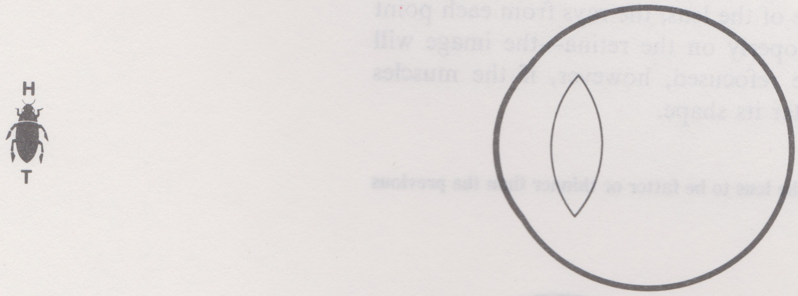


Figure 58

Along the central axis of the eye, the two faces of the lens and the surface of the cornea are parallel. When a ray passes through parallel faces you have already seen that it goes on undeviated (it will be slightly displaced laterally, but you need not worry unduly about that). So if one were to draw a ray passing from a point on the insect's head, H, and through a point somewhere between the cornea and the back face of the lens, it would carry on to the retina undeviated. This serves to locate the position of the image of the head, H′ (Fig. 59). The same procedure serves to locate the image T′ of the tail, T. As you can see the image turns out to be inverted (the reason one mentally 'sees' the image the right way up is to do with the way the brain interprets the signals coming from the retina).

If the shape of the lens is such that the object is correctly focused on the retina one can now draw in any rays one pleases. We show in Figure 59 two further rays coming from the head and converging on the same point of intersection with the retina as our earlier ray. We also show two additional rays coming from the tail.*

* *You will notice in this diagram that we have drawn the rays as though they change direction inside the lens, this of course, does not happen; refraction takes place at the two surfaces of the lens and it is these two changes in direction that give the combined overall deviation. However it is a convention when representing rays passing through a lens, to extrapolate the incident and emergent rays to meet as shown. From now on we shall adopt this convention.*

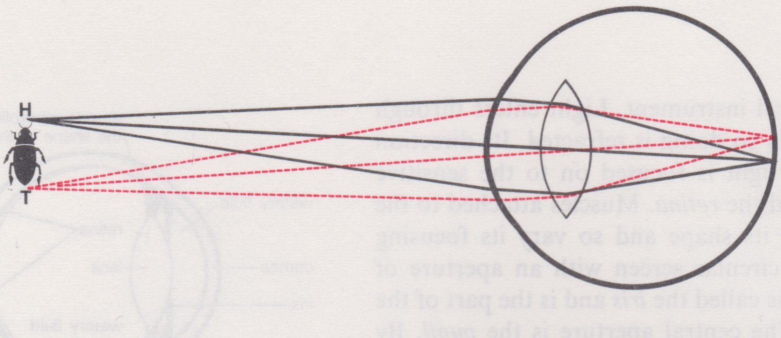


Figure 59

Diagram showing how an image is formed on the retina of the eye.

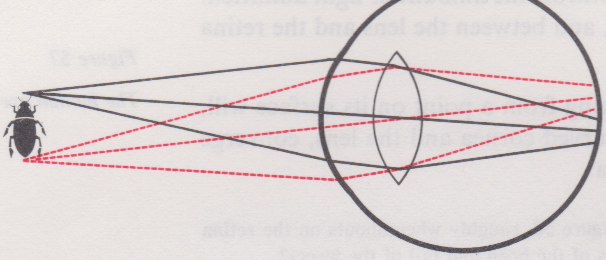


Figure 60

Diagram showing that when an object is brought closer to the eye and the shape of the lens remains unaltered, the image is no longer correctly focused.

If the object is now brought closer to the eye, as in Figure 60, without making any adjustment to the shape of the lens, the rays from each point on the object will not converge properly on the retina—the image will appear blurred. The image can be refocused, however, if the muscles attached to the lens are made to alter its shape.

Do you expect the new shape of the lens to be fatter or thinner than the previous one?

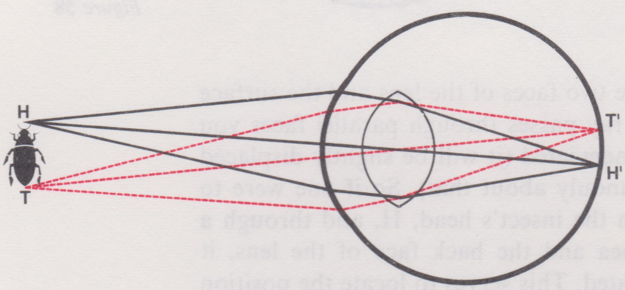


Figure 61

Diagram showing the lens altered in shape in order to focus the image with the object in its new position.

Figure 61, shows the shape of the lens after this refocusing has been done. You will note that in Figure 61 the image is bigger on the retina than in Figure 59 when the object was further away. You might think then that in order to see smaller and smaller objects all one has to do is to bring them up closer.

But what do you find happens when you bring an object up very, very close to your eye? Try it.

There is a limit to the amount by which the lens of the eye can alter its shape. At distances of little less than that at which you are reading this print, your eyes can no longer be made to bend the rays sufficiently to secure good focus—you get the kind of situation you saw earlier in Figure 60. This inability tends to become more marked with advancing age.

Appendix 6 (Black)

Exponential Functions

Look at the working of the answer to the problem following equation 3 (p. 41): you will note that instead of writing $N_t = 1\,600\,(\frac{1}{2})^{12/3}$, we could equally well have written $N_t = 1\,600\,(\frac{1}{4})^{12/6}$ or $1\,600\,(\frac{1}{16})^{12/12}$. This means we are not restricted to expressing the speed of the reaction by a half-life; if we so chose we could express it in terms of 'a quarter-life', i.e. the time for the initial number of nuclei to be reduced to a quarter (6 hours), or as 'a sixteenth-life' (12 hours). In practice, one never uses these particular terms, but they do serve to illustrate something important—there are an infinite number of ways of specifying the rate of the transformation. We can express it in a general way in terms of a time interval T_p, such that a fraction p of the original sample remains at the end of this period. Then

$$N_t = Np^{\,t/T_p} \qquad \text{(7)}$$

Make sure that you understand what each of these symbols stands for. If you have forgotten any, refer back to p. 41, bearing in mind that there p was taken to be $\frac{1}{2}$.

It is customary to use the half-life and also one other value of T_p. This second value is called the *mean-life*, τ (a Greek letter pronounced 'taw'). The mean life for a reaction is the average time that a nucleus lives before decaying. It is obtained by adding together the individual times that each of the N X nuclei lived, $t_1, t_2, t_3 \ldots t_N$, and dividing by the total number of X nuclei, i.e.

$$\tau = \frac{t_1 + t_2 + t_3 \ldots t_N}{N} \qquad \text{(8)}$$

Assuming we wish to express N_t in terms of the mean-life, the question naturally arises as to what value does p assume in equation (7) for such a value of T_p, i.e. what fraction of the X nuclei remain after time, τ? Taking the mathematical form of the curve in Figure 36 (p. 41) and using calculus one finds the answer to be

$$p = \frac{1}{2.718\ldots}$$

The denominator is an example of an irrational number (it continues to an infinite number of decimal places). It is a number that appears in a great many scientific problems and because of its importance it is assigned a special symbol, e (this is similar to the way we assign the special symbol, π, to another irrational number, 3.1415 . . ., calculated from the ratio of the circumference to the diameter of a circle).

Thus, $\quad N_t = N\left(\frac{1}{e}\right)^{t/\tau}$

or inverting, $\quad N_t = N\,e^{-t/\tau} \qquad \text{(9)}$

The fraction of nuclei remaining after time, τ, i.e. $\left(\frac{1}{e}\right)$, is 0.368. Thus, fewer than half of the nuclei remain; the mean-life is therefore greater than the half-life. How much greater?

Rearranging equation (9):

$$\frac{N}{N_t} = e^{t/\tau}$$

At time $t=T_{\frac{1}{2}}$,

$$N_t=\tfrac{1}{2}N$$

$$\text{Thus} \quad N/\tfrac{1}{2}N=e^{T_{\frac{1}{2}}/\tau}$$

$$\text{i.e. } 2=e^{T_{\frac{1}{2}}/\tau}$$

Taking logarithms of both sides

$$\log_{10} 2=(T_{\frac{1}{2}}/\tau)\log_{10} e$$
$$=(T_{\frac{1}{2}}/\tau)\log_{10} 2.718$$
$$0.3010=0.4343T_{\frac{1}{2}}\tau$$
$$T_{\frac{1}{2}}=\frac{0.3010}{0.4343}\tau$$
$$T_{\frac{1}{2}}=0.693\tau \quad\text{.......................................}(10)$$

This gives us the relationship between the half-life, $T_{\frac{1}{2}}$, and the mean-life, τ.

Normally the speed of a radioactive decay is characterized by its half-life. In the decays of sub-nuclear particles however it is customary to use the mean-life. The mean-life is particularly useful as its inverse is the *decay constant*, λ, of the reaction,

$$\frac{1}{\tau}=\lambda \quad\text{.......................................}(11)$$

The decay constant is the constant of proportionality relating the number of nuclei decaying per unit time to the number of nuclei present, i.e.

$$-\frac{dN}{dt}=\lambda N$$

$$\text{or} \quad -\frac{dN}{N}=\lambda dt \quad\text{.......................................}(12)$$

Where dN denotes 'a small change in N', and dt is 'a small change in t'. (The letter 'd' stands for 'differential'.) The negative sign indicates the change in N is a decrease.

If you look at the answer to the problem following equation (5) (p. 43) you see that there is only a comparatively small change in N (just over 2 per cent, so you should be able immediately to work out the decay constant for the reaction:

$$^{87}Rb \rightarrow {}^{87}Sr$$

Try it.

$$-\frac{dN}{N}=\frac{0.015}{0.680+0.015}$$
$$\approx 0.022$$
$$dt=1.5\times 10^9 \text{ years}$$
$$\therefore \lambda \approx \frac{0.022}{1.5\times 10^9}=1.5\times 10^{-11}/\text{year}$$

From this value of λ now obtain a value for the mean life-time, τ:

$$\tau=\frac{1}{\lambda}$$
$$\tau=6.7\times 10^{10} \text{ years}$$

From the relation between $T_{\frac{1}{2}}$ and τ given earlier, equation (10), we can check our result for τ

$$T_{\frac{1}{2}}=0.693\,\tau$$
$$\therefore \tau=\frac{4.7\times 10^{10}}{0.693}=6.8\times 10^{10} \text{ years}$$

This agrees within the accuracy of our calculations.

Appendix 7 (White)

Glossary

CYCLE Any series of changes or operations performed by or on a system which brings it back to its original state. E.g., a vibrating object executes one cycle when it travels from one extreme position to the opposite and back again.

GREAT CIRCLE Circle obtained by cutting a sphere by a plane passing through the centre. e.g., regarding the Earth as a sphere, the equator is a great circle, as are all meridians of longitude.

INTEGRAL MULTIPLE A multiplying factor consisting of a whole number i.e. integer.

INTENSITY Generally means amount. e.g., 'intensity of electron beam' means the amount or number of electrons contained in the beam.

MEAN Generally understood to be the arithmetic mean, i.e. the average.

NORMAL A line that is perpendicular to a surface.

ORGANIC CHEMICAL SUBSTANCES Chemical substances containing carbon combined with hydrogen and often other elements.

STANDARD ATMOSPHERIC PRESSURE A unit of pressure. The pressure that will support a column of mercury 760 mm high at 0° C, sea-level and latitude 45°. (Atmospheric pressure fluctuates about this value from day to day.)

SUBTEND Two points, A and B, are said to subtend the angle ACB at the point C.

Self-Assessment Questions

Sections 2.1 and 2.2

Question 1 (*Objective 1*)

	(a) true	(b) false
The wavelength of a wave is the reciprocal of the frequency	☐	☐

Question 2 (*Objective 1*)

	(a) true	(b) false
The velocity of a wave is the product of the frequency and the wavelength	☐	☐

Question 3 (*Objective 1*)

	(a) true	(b) false
Ultrasonic waves are sound waves that are too weak to be heard	☐	☐

Question 4 (*Objective 1*)

	(a) true	(b) false
A sound wave with frequency 30 000 Hz is an example of an ultrasonic wave	☐	☐

Question 5 (*Objective 1*)

	(a) true	(b) false
The amplitude of a sound wave is the maximum speed with which the particles of the medium move as they oscillate about their mean position	☐	☐

Question 6 (*Objective 14*)

When a wave moves into a medium where its velocity is lower,

	(a) increases	(b) decreases	(c) remains unchanged
(i) the wavelength	☐	☐	☐
(ii) the frequency	☐	☐	☐

Section 2.3

Question 7 (*Objective 4*)

Which type of radiation would be used to investigate each of the problems listed below. Tick the appropriate box.

(a) —ultrasonic
(b) —infra-red
(c) —ultra-violet light
(d) —X-rays

	(a)	(b)	(c)	(d)
(i) A document suspected of being fraudulently altered	☐	☐	☐	☐
(ii) The whereabouts of a shoal of fish	☐	☐	☐	☐
(iii) Whether a bone has been fractured	☐	☐	☐	☐
(iv) The successive alterations made to a painting	☐	☐	☐	☐
(v) The area over which a crop has become diseased	☐	☐	☐	☐
(vi) The whereabouts of an army convoy at night	☐	☐	☐	☐

Question 8 (*Objective 5*)

Electromagnetic radiation of the following wavelengths (in metres) are classed as what type of radiation listed on the left? Tick the appropriate box.

	(a) 10^{-4}	(b) 5×10^{-7}	(c) 10^{-6}	(d) 3×10^{-7}	(e) 10^{3}
(i) visible	☐	☐	☐	☐	☐
(ii) ultra-violet	☐	☐	☐	☐	☐
(iii) gamma	☐	☐	☐	☐	☐
(iv) X-ray	☐	☐	☐	☐	☐
(v) infra-red	☐	☐	☐	☐	☐
(vi) radio and TV	☐	☐	☐	☐	☐

Question 9 (*Objective 6*)

When two materials are rubbed together the one that has acquired a positive charge has:

(i) gained nuclei ☐
(ii) gained electrons ☐
(iii) lost nuclei ☐
(iv) lost electrons ☐

Question 10 (*Objective 7*)

Tick the appropriate boxes

	(a) ultra-sonic waves	(b) ultra-violet waves
(i) Require a medium	☐	☐
(ii) The source must move in order to produce the wave	☐	☐
(iii) The quantity that is varying in the wave is directed along the ray representing the path of the wave	☐	☐
(iv) Can exhibit diffraction	☐	☐

Question 11 (*Objective 11*)

The electric force on a small charged body, B, produced by another stationary charged body, A, acts

(i) along the direction AB ☐
(ii) at right-angles to AB ☐
(iii) in some other direction ☐

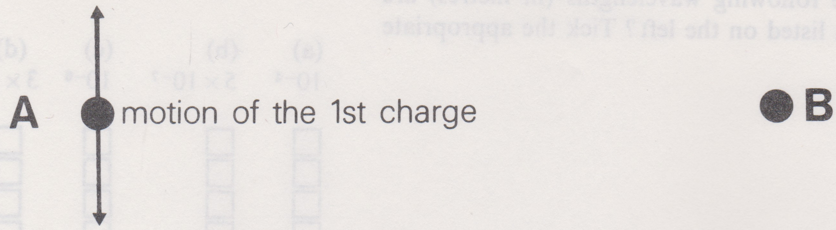
If body A oscillates back and forth in the direction along the line AB, the field at B acts

(iv) along the direction AB ☐
(v) at right-angles to AB ☐
(vi) in some other direction ☐

Question 12 (*Objective 11*)

A positive electric charge, A, oscillates about its mean position as shown in the diagram. Explain briefly what will happen to a second positive charge, B, initially at rest, if this second charge is set free so as to be able

to move in any direction. What effect if any would you expect the presence of the second charge to have on the first charge?



Section 2.4

Question 13 (*Objective 1*)

To obtain a line spectrum light of a single wavelength is needed

(a) true ☐ (b) false ☐

Question 14 (*Objectives 1, 10*)

The standard metre is defined as a length equal to a certain number of wavelengths, measured in air at standard pressure, of a particular reddish-orange line in the spectrum of the gas krypton

(a) true ☐ (b) false ☐

Question 15 (*Objective 9*)

All optical illusions, including after-images, arise from the way the brain organizes the information sent to it

(a) true ☐ (b) false ☐

Section 2.5

Question 16 (*Objective 1*)

A battery is a device for maintaining an electric field around a circuit

(a) true ☐ (b) false ☐

Question 17 (*Objective 1*)

An electron microscope is so-called because it is a microscope for studying electrons

(a) true ☐ (b) false ☐

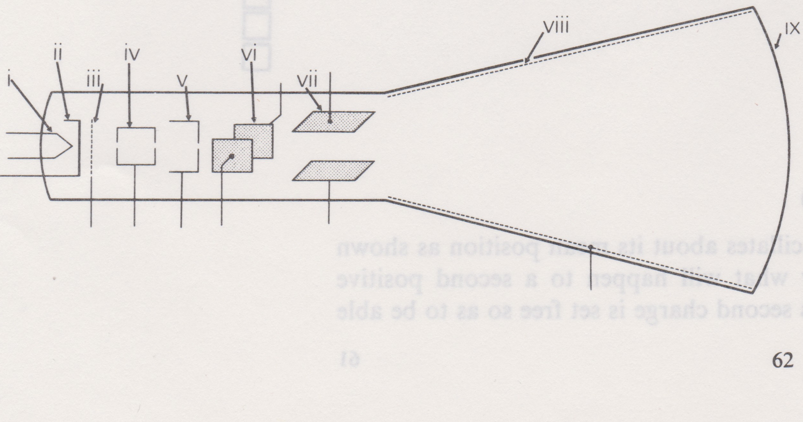
Question 18 (*Objective 1*)

In the triangulation method of distance estimation, the baseline is the distance from the observer to the object being viewed

(a) true ☐ (b) false ☐

Question 19 (*Objective 2*)

Name the parts of the cathode ray tube:



(i)
(ii)
(iii)
(iv)
(v)
(vi)
(vii)
(viii)
(ix)

Question 20 (*Objective 12*)

The following effects all play a part in the production of a beam of free electrons. Place numbers 1 to 7 in the appropriate boxes to indicate the order in which the effects take place

(i) increase in random motion of electrons ☐
(ii) systematic motion of electrons established in a circuit ☐
(iii) electrons focused by focusing anode ☐
(iv) chemical reaction set up in a battery ☐
(v) electrons pass through the accelerating anode ☐
(vi) electrons escape from a metal surface ☐
(vii) electrons collide with a lattice of atoms ☐

Question 21 (*Objective 13*)

The law of conservation of electric charge states that the total amount of positive charge equals the total amount of negative charge

(a) true ☐ (b) false ☐

Question 22 (*Objective 14*)

A tank has a hole in the bottom out of which water is allowed to flow. The height of the water level in the tank drops from 16 cm to 4 cm in 56 seconds. Assuming the rate at which the water leaves is proportional to the height of the water level, calculate how long it would take for the level to drop from 40 cm to 2.5 cm.

Appendix 1

Question 23 (*Objective 8*)

Which items chosen from the right-hand column are associated with those on the left? Indicate your answers below:

(i) Fixer.
(ii) Migration of electrons.
(iii) Emulsion.
(iv) Celluloid.
(v) Developer.

(a) A mixture of silver bromide crystals and gelatine.
(b) Formation of latent image.
(c) Removal of silver bromide.
(d) Deposition of silver on a large scale.
(e) A supporting base.

(i)
(ii)
(iii)
(iv)
(v)

Appendix 2

Question 24 (*Objective 1*)

The objective lens of a microscope is the lens closest to the object

(a) true ☐ (b) false ☐

Question 25 (*Objective 14*)

A man six feet tall is photographed at a distance 18 ft from the camera, and his image on the film is found to measure 1 inch.

What is the distance between the lens and the film?

Appendix 5

Question 26 (*Objective 1*)

The iris controls the amount of light admitted into the human eye

(a) true ☐ (b) false ☐

Question 27 (*Objective 3*)

The speed of light in the cornea of the eye is about three-quarters that in air. In the watery fluids the speed is about the same as in the cornea. Its mean value in the lens is somewhat lower. At which boundaries would you expect refraction to take place?

Answer by ticking the appropriate box or boxes.

(i) air—cornea	☐	☐
(ii) cornea—first watery fluid	☐	☐
(iii) first watery fluid—lens	☐	☐
(iv) lens—second watery fluid	☐	☐

The Handling of Experimental Data, Section 3.1.5

Question 28 (*Objective 1*)

One-millionth of 10^{-3} second is a:

Tick correct answer

(i) millisecond ☐
(ii) microsecond ☐
(iii) nanosecond ☐
(iv) none of these ☐

Self-Assessment Answers and Comments

Question 1

Answer (b)

Comment $= v/f$ *not* $1/f$

Question 2

Answer (a) $v = \lambda f$

Question 3

Answer (b)

Comment

Ultrasonic waves cannot be heard because of their high frequency, not because they are necessarily of low intensity.

Question 4

Answer (a)

Question 5

Answer (b)

Comment

The amplitude is the maximum distance a particle of the medium moves from its mean position.

Question 6

Answer (i) b, (ii) c.

Question 7

Answer (i) c, (ii) a, (iii) (d), (iv) c, (v) b, (vi) b.

Comment

With regard to the last item it has been found that the warmth from the engines of the army trucks makes them visible in the infra-red region. Anything that is warm is liable to show up well in the infra-red. You might also have included (iv)d because X-rays can also be used to examine paintings.

Question 8

Answer (v) a, (i) b, (v) c, (ii) d, (vi) e.

Comment

There were no wavelengths specified corresponding to gamma or X-rays.

Question 9

Answer (iv)

Question 10

Answer (i) a, (ii) a and b, (iii) a, (iv) a and b.

Question 11

Answer (i) and (iv)

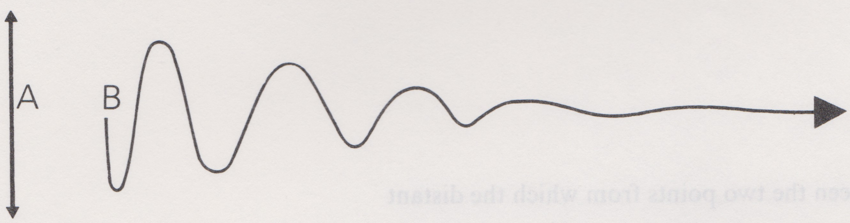
Comment

If you chose (v) instead of (iv) you should note that we are here considering the charge to move *in* the direction of interest, not at right-angles as was the case in the text. Because of this there can be no sideways component of the electric field. The field will always be directed along AB. The effect of varying the distance of A from B will be to vary the strength of the field, not its direction.

Question 12

Answer

B experiences a steady component of force acting directly away from the mean position. It also experiences a fluctuating component at right-angles to this direction. It will therefore move away from A along a path that wanders from side to side as in the diagram.



You might also add that the amplitude of the up and down movements diminishes the further away B gets from A because the strength of the field diminishes with distance. Also the charge moves with ever increasing speed.

Whatever force is exerted by A on B, there will be an equal and opposite force by B on A. Just as B is regarded as being acted upon by the electric field of A, so A is acted upon by the electric field of B. As B moves up and down in response to the up and down movement of A, so A will experience an up and down force due to the movement of B. As B is repelled by A so A is repelled by B.

Question 13

Answer (b)

Comment

Light of a single wavelength is one example of a line spectrum, but generally speaking there is more than one wavelength.

Question 14

Answer (b)

Comment

In vacuum, not in air.

Question 15

Answer (b)

Comment

An after-image is caused by fatigue of the sensory receptors rather than by the manner in which the brain organizes the information sent to it.

Question 16

Answer (a)

Question 17

Answer (b)

Comment

The electron microscope is so called because it *uses* electrons, not because it is designed for studying electrons.

Question 18

Answer (b)

Comment

The baseline is the distance between the two points from which the distant object is viewed.

Question 19

Answer See Figure 40 of main text.

Question 20

Answer (i) 4 (iii) 6 (v) 7 (vii) 3
(ii) 2 (iv) 1 (vi) 5

Question 21

Answer (b)

Comment

The law says that the amount of positive charge less the amount of negative charge must remain constant; it does not say that it must be zero.

Question 22

Answer 112 seconds

4 cm is a quarter of 16 cm, therefore 56 seconds represents twice the 'half-life' for the height of the water level. The half-life is thus 28 seconds. Starting at 40 cm, each successive half-life leaves a level of 20, 10, 5 and 2.5 cm. Therefore to go from 40 to 2.5 cm requires 4 half-lives, i.e. $4 \times 28 =$ 112 seconds.

Question 23

Answer (i) c, (ii) b, (iii) a, (iv) e, (v) d.

Question 24

Answer (a)

Question 25

Answer 3 inches.

$$\frac{\text{size of image}}{\text{size of object}} = \frac{\text{distance of image to lens}}{\text{distance of object to lens}}$$

$$\frac{1 \text{ inch}}{(6 \times 12) \text{ inches}} = \frac{\text{distance of image to lens}}{(18 \times 12) \text{ inches}}$$

$$\text{distance of image to lens} = \frac{(18 \times 12) \times 1 \text{ inch}}{(6 \times 12)}$$

$$= 3 \times 1 \text{ inch}$$

Question 26

Answer (a)

Comment

The amount of light admitted to the eye is governed by the size of the pupil—but this is controlled by the iris.

Question 27

Answer (i), (iii) and (iv).

Comment

Refraction occurs only where there is a change of speed.

Question 28

Answer (iii)

Acknowledgements

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P. W. ALLEN AND COMPANY, Fig. 13; CHARLES F. BRANDFORD CO., PUBLISHERS, Newton, Mass., U.S.A., Fig. 38; C. B. DAISH, Fig. 37; DOVER PUBLICATIONS INC., from *Visual Illusions* by M. Luckiesh, Fig. 17; ELEK BOOKS LTD., from *The New Age in Physics* by Sir Harry Massey, Fig. 34; LABORATOIRE D'OPTIQUE, C.N.R.S., Fig. 33; FRANK W. LANE, Fig. 3; M. J. LANGFORD and THE FOCAL PRESS LTD., Fig. 43; MCGRAW HILL BOOK CO., from *Principles of Optics* by Hardy and Perrin, Fig. 25; NATIONAL AGRICULTURAL ADVISORY SERVICE, E. Region, Ministry of Agriculture, Crown Copyright, Fig. 14; NATIONAL PHYSICAL LABORATORY, Teddington, Crown Copyright, Fig. 20; ST. BARTHOLOMEW'S HOSPITAL AND MEDICAL COLLEGE, Fig. 15; ST. THOMAS'S HOSPITAL MEDICAL SCHOOL, Fig. 12; U.S. ATOMIC ENERGY COMMISSION, Fig. 39; SCIENTIFIC AMERICAN INC., from 'Standards of Measurement' by A. V. Astin, *Scientific American*, June 1968, Copyright © 1968 Scientific American, Inc., Fig. 18.

Notes

S.100—SCIENCE FOUNDATION COURSE UNITS

1 Science: Its Origins, Scales and Limitations
2 Observation and Measurement

3 Mass, Length and Time
4 Forces, Fields and Energy

5 The States of Matter

6 Atoms, Elements and Isotopes: Atomic Structure
7 The Electronic Structure of Atoms

8 The Periodic Table and Chemical Bonding
9 Ions in Solution

10 Covalent Compounds

11, 12 Chemical Reactions

13 Giant Molecules

14 The Chemistry and Structure of the Cell

15, 16 Cell Dynamics and the Control of Cellular Activity

17 The Genetic Code: Growth and Replication
18 Cells and Organisms

19 Evolution by Natural Selection
20 Species and Populations

21 Unity and Diversity

22 The Earth: Its Shape, Internal Structure and Composition

23 The Earth's Magnetic Field

24 Major Features of the Earth's Surface
25 Continental-Movement, Sea-floor Spreading and Plate Tectonics

26, 27 Earth History

28 The Wave Nature of Light

29 Quantum Theory
30 Quantum Physics and the Atom

31 The Nucleus of the Atom
32 Elementary Particles

33, 34 Science and Society